Basic Direct Current (DC) Theory

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DC SOURCES

When most people think of DC, they usually think of batteries. In addition to batteries, however, there are other devices that produce DC which are frequently used in modern technology.

1.1 LIST the four ways to produce a DC voltage.

1.2 STATE the purpose of a rectifier.

1.3 DESCRIBE the outputs of the following circuits:
a. Half-wave bridge rectifier
b. Full-wave bridge rectifier

Batteries

A battery consists of two or more chemical cells connected in series. The combination of materials within a battery is used for the purpose of converting chemical energy into electrical energy. To understand how a battery works, we must first discuss the chemical cell.

The chemical cell is composed of two electrodes made of different types of metal or metallic compounds which are immersed in an electrolyte solution. The chemical actions which result are complicated, and they vary with the type of material used in cell construction. Some knowledge of the basic action of a simple cell will be helpful in understanding the operation of a chemical cell in general.

In the cell, electrolyte ionizes to produce positive and negative ions (Figure 1, Part A). Simultaneously, chemical action causes the atoms within one of the electrodes to ionize.

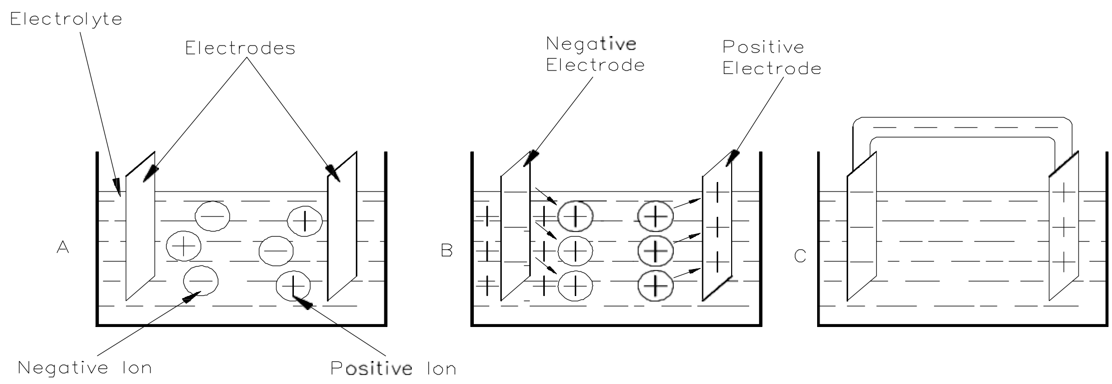

Figure 1 Basic Chemical Battery

Due to this action, electrons are deposited on the electrode, and positive ions from the electrode pass into the electrolyte solution (Part B). This causes a negative charge on the electrode and leaves a positive charge in the area near the electrode (Part C).

The positive ions, which were produced by ionization of the electrolyte, are repelled to the other electrode. At this electrode, these ions will combine with the electrons. Because this action causes removal of electrons from the electrode, it becomes positively charged.

DC Generator

A simple DC generator consists of an armature coil with a single turn of wire. The armature coil cuts across the magnetic field to produce a voltage output. As long as a complete path is present, current will flow through the circuit in the direction shown by the arrows in Figure 2. In this coil position, commutator segment 1 contacts with brush 1, while commutator segment 2 is in contact with brush 2.

Rotating the armature one-half turn in the clockwise direction causes the contacts between the commutator segments to be reversed. Now segment 1 is contacted by brush 2, and segment 2 is in contact with brush 1.

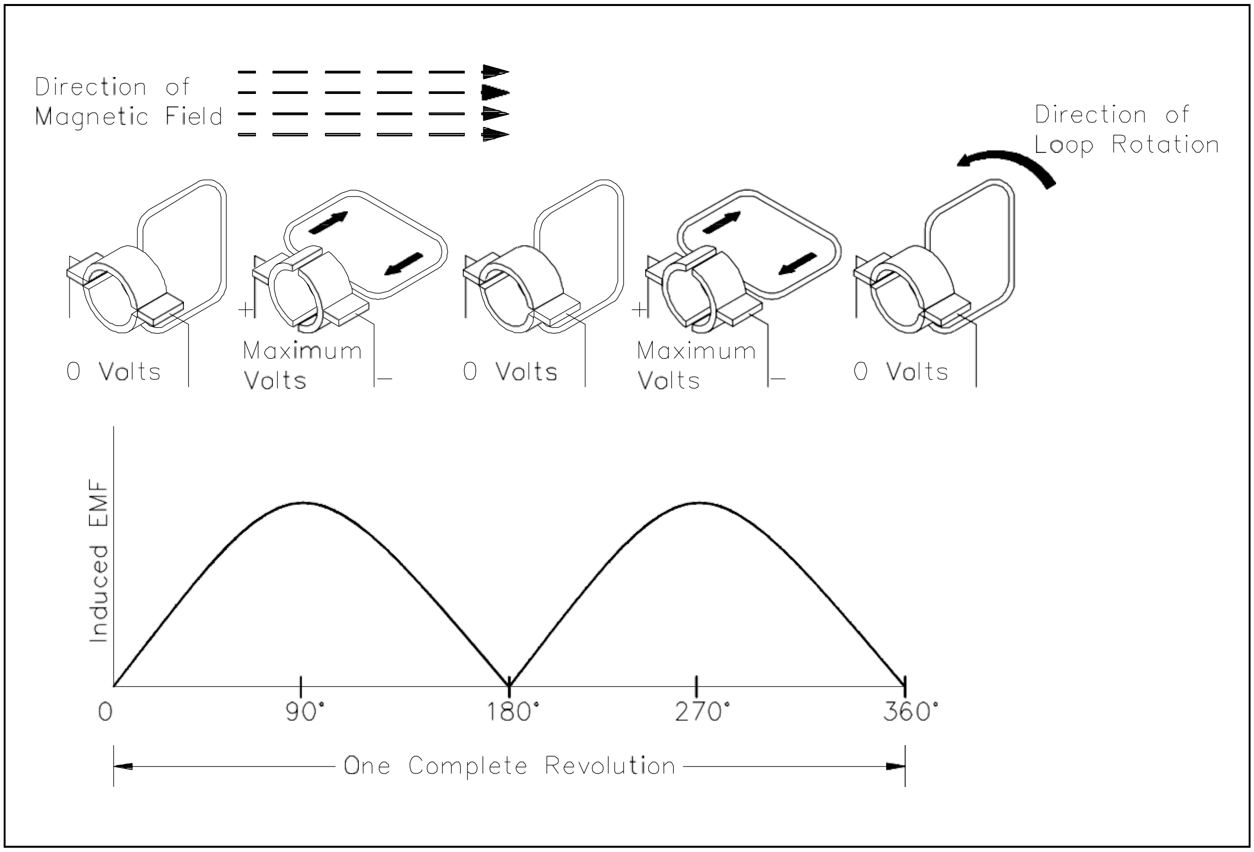

Figure 2 Basic DC Generator

Due to this commutator action, that side of the armature coil which is in contact with either of the brushes is always cutting the magnetic field in the same direction. Brushes 1 and 2 have a constant polarity, and pulsating DC is delivered to the load circuit.

Thermocouples

A thermocouple is a device used to convert heat energy into a voltage output. The thermocouple consists of two different types of metal joined at a junction (Figure 3).

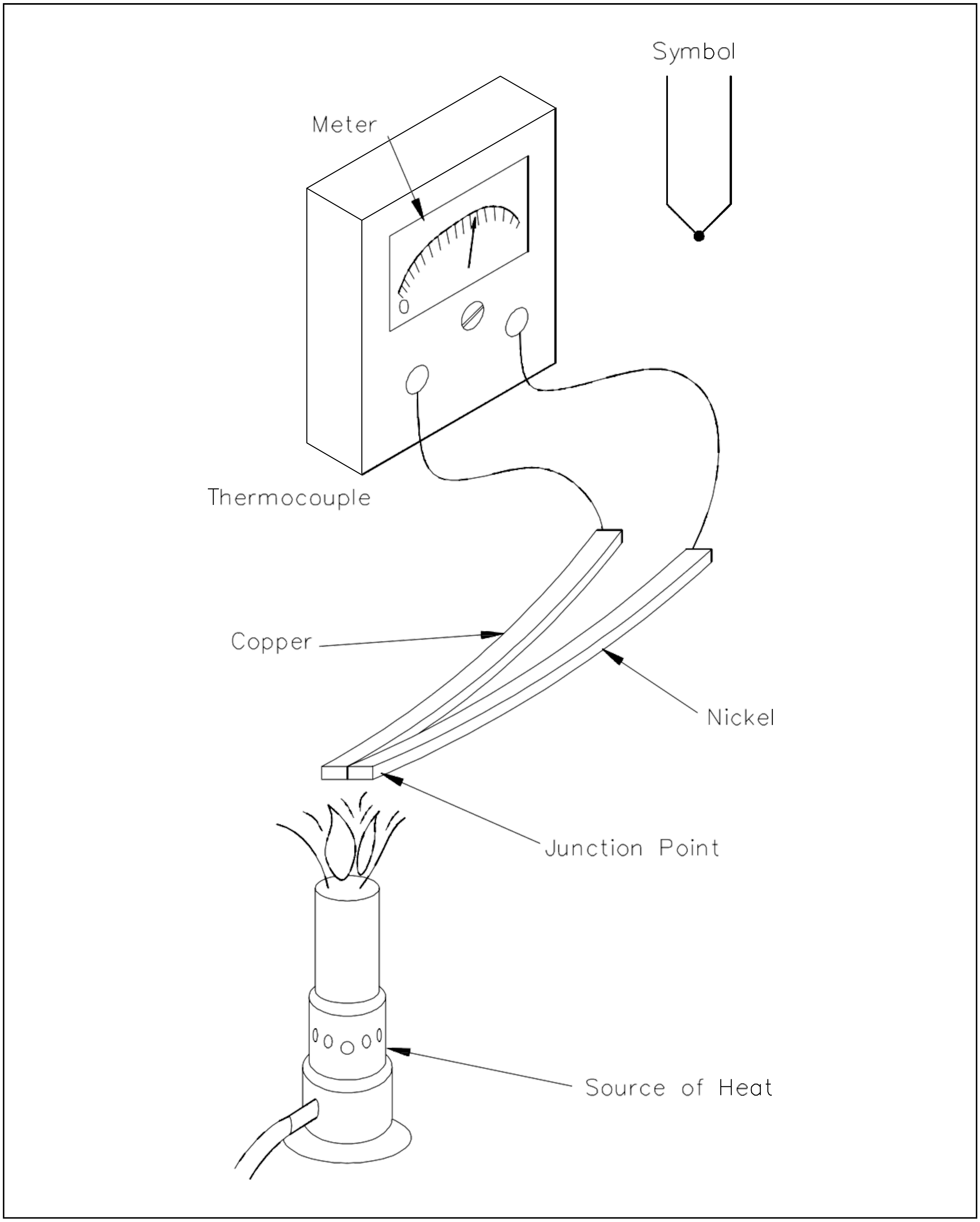

Figure 3 Production of a DC Voltage Using a Thermocouple

As the junction is heated, the electrons in one of the metals gain enough energy to become free electrons. The free electrons will then migrate across the junction and into the other metal. This displacement of electrons produces a voltage across the terminals of the thermocouple. The combinations used in the makeup of a thermocouple include: iron and constantan; copper and constantan; antimony and bismuth; and chromel and alumel.

Thermocouples are normally used to measure temperature. The voltage produced causes a current to flow through a meter, which is calibrated to indicate temperature.

Rectifiers

Most electrical power generating stations produce alternating current. The major reason for generating AC is that it can be transferred over long distances with fewer losses than DC; however, many of the devices which are used today operate only, or more efficiently, with DC. For example, transistors, electron tubes, and certain electronic control devices require DC for operation. If we are to operate these devices from ordinary AC outlet receptacles, they must be equipped with rectifier units to convert AC to DC. In order to accomplish this conversion, we use diodes in rectifier circuits. The purpose of a rectifier circuit is to convert AC power to DC.

The most common type of solid state diode rectifier is made of silicon. The diode acts as a gate, which allows current to pass in one direction and blocks current in the other direction. The polarity of the applied voltage determines if the diode will conduct. The two polarities are known as forward bias and reverse bias.

Forward Bias

A diode is forward biased when the positive terminal of a voltage source is connected to its anode, and the negative terminal is connected to the cathode (Figure 4A). The power source's positive side will tend to repel the holes in the p-type material toward the p-n junction by the negative side. A hole is a vacancy in the electron structure of a material. Holes behave as positive charges. As the holes and the electrons reach the p-n junction, some of them break through it (Figure 4B). Holes combine with electrons in the n-type material, and electrons combine with holes in the p-type material.

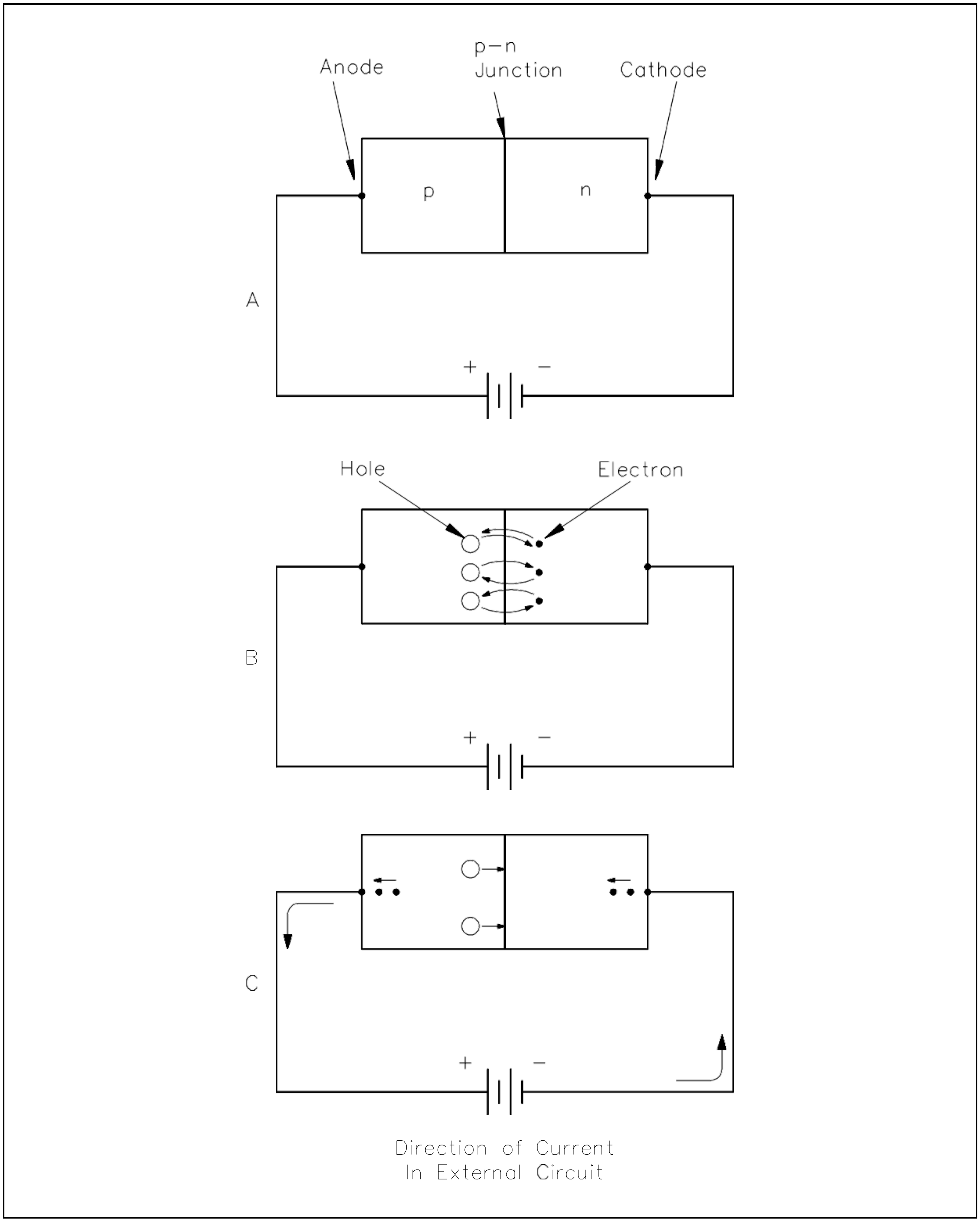

Figure 4 Forward-Biased Diode

When a hole combines with an electron, or an electron combines with a hole near the p-n junction, an electron from an electron-pair bond in the p-type material breaks its bond and enters the positive side of the source. Simultaneously, an electron from the negative side of the source enters the n-type material (Figure 4C). This produces a flow of electrons in the circuit.

Reverse Bias

Reverse biasing occurs when the diode's anode is connected to the negative side of the source, and the cathode is connected to the positive side of the source (Figure 5A). Holes within the p-type material are attracted toward the negative terminal, and the electrons in the n-type material are attracted to the positive terminal (Figure 5B). This prevents the combination of electrons and holes near the p-n junction, and therefore causes a high resistance to current flow. This resistance prevents current flow through the circuit.

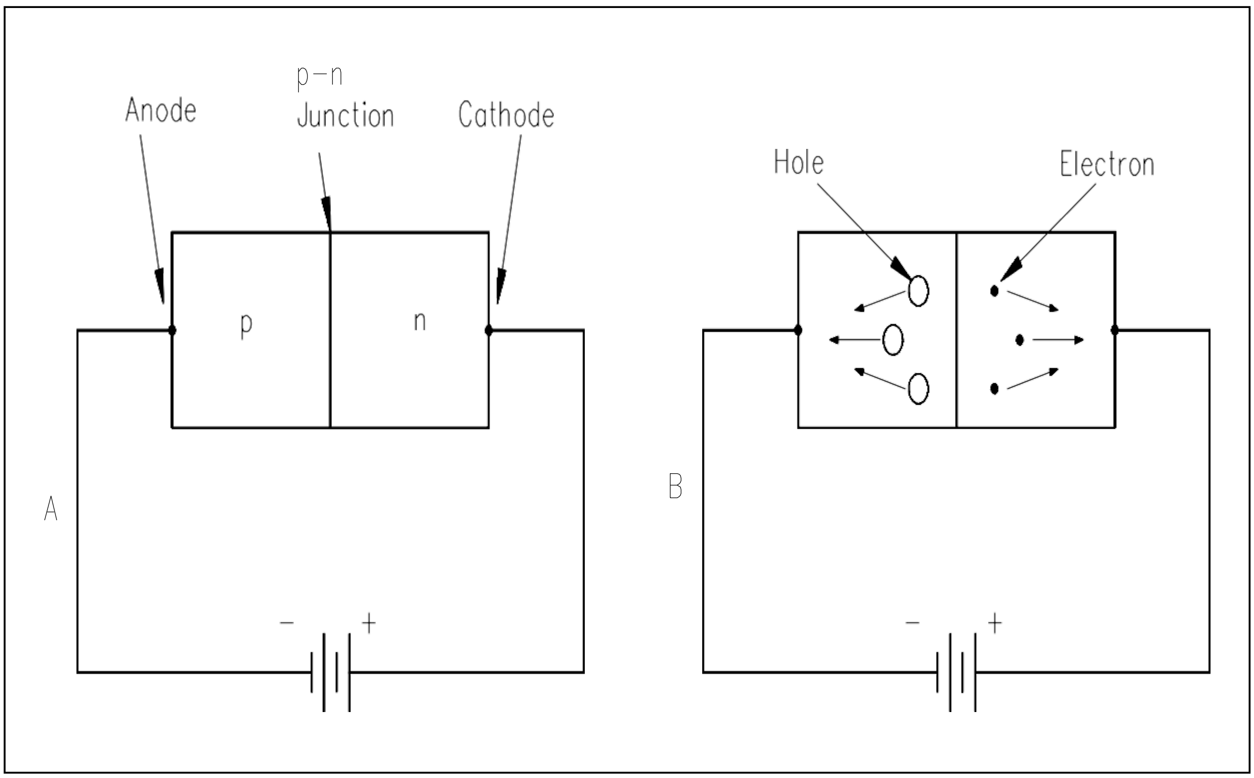

Figure 5 Reverse-Biased Diode

Half-Wave Rectifier Circuit

When a diode is connected to a source of alternating voltage, it will be alternately forward-biased, and then reverse-biased, during each cycle of the AC sine-wave. When a single diode is used in a rectifier circuit, current will flow through the circuit only during one-half of the input voltage cycle (Figure 6). For this reason, this rectifier circuit is called a half-wave rectifier. The output of a half-wave rectifier circuit is pulsating DC.

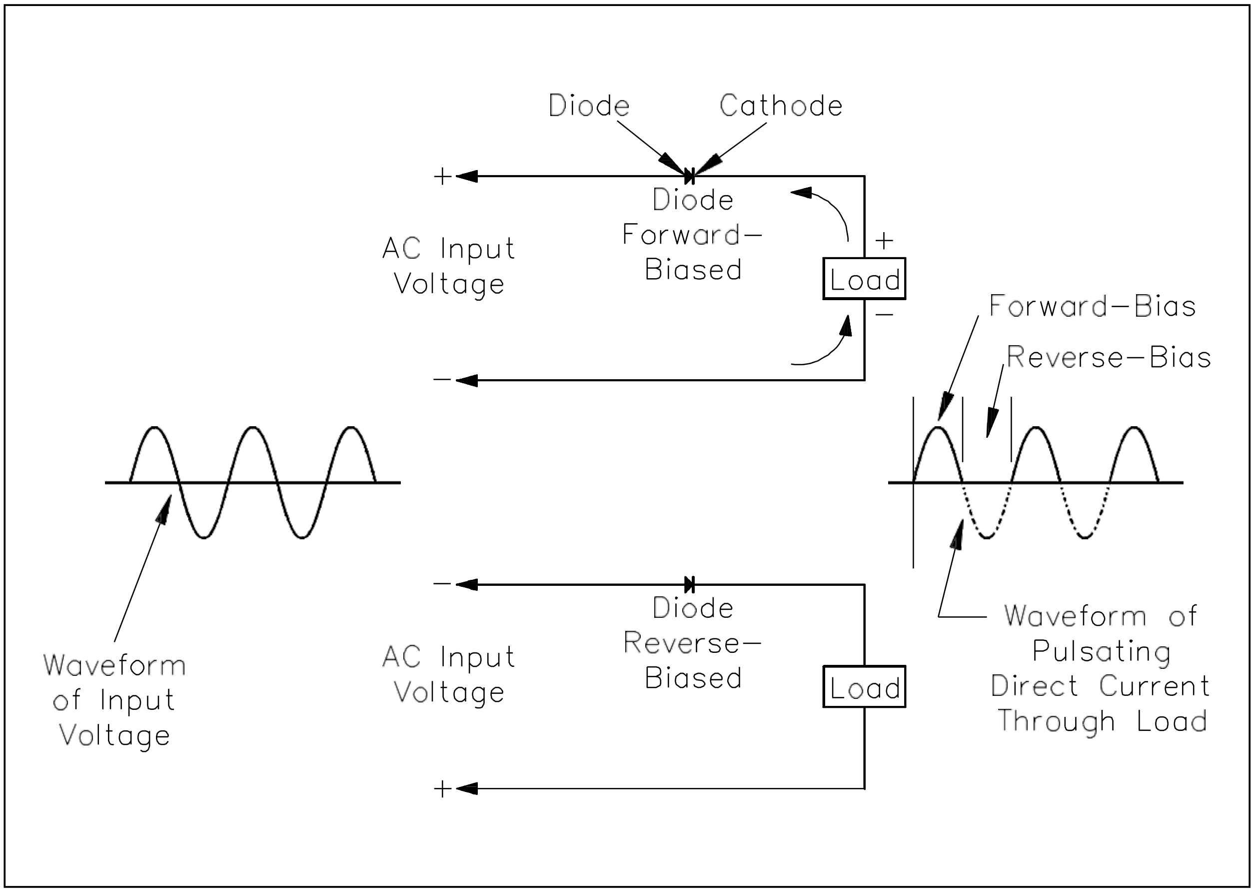

Figure 6 Half-Wave Rectifier

Full-Wave Rectifier Circuit

A full-wave rectifier circuit is a circuit that rectifies the entire cycle of the AC sine-wave. A basic full-wave rectifier uses two diodes. The action of these diodes during each half cycle is shown in Figure 7.

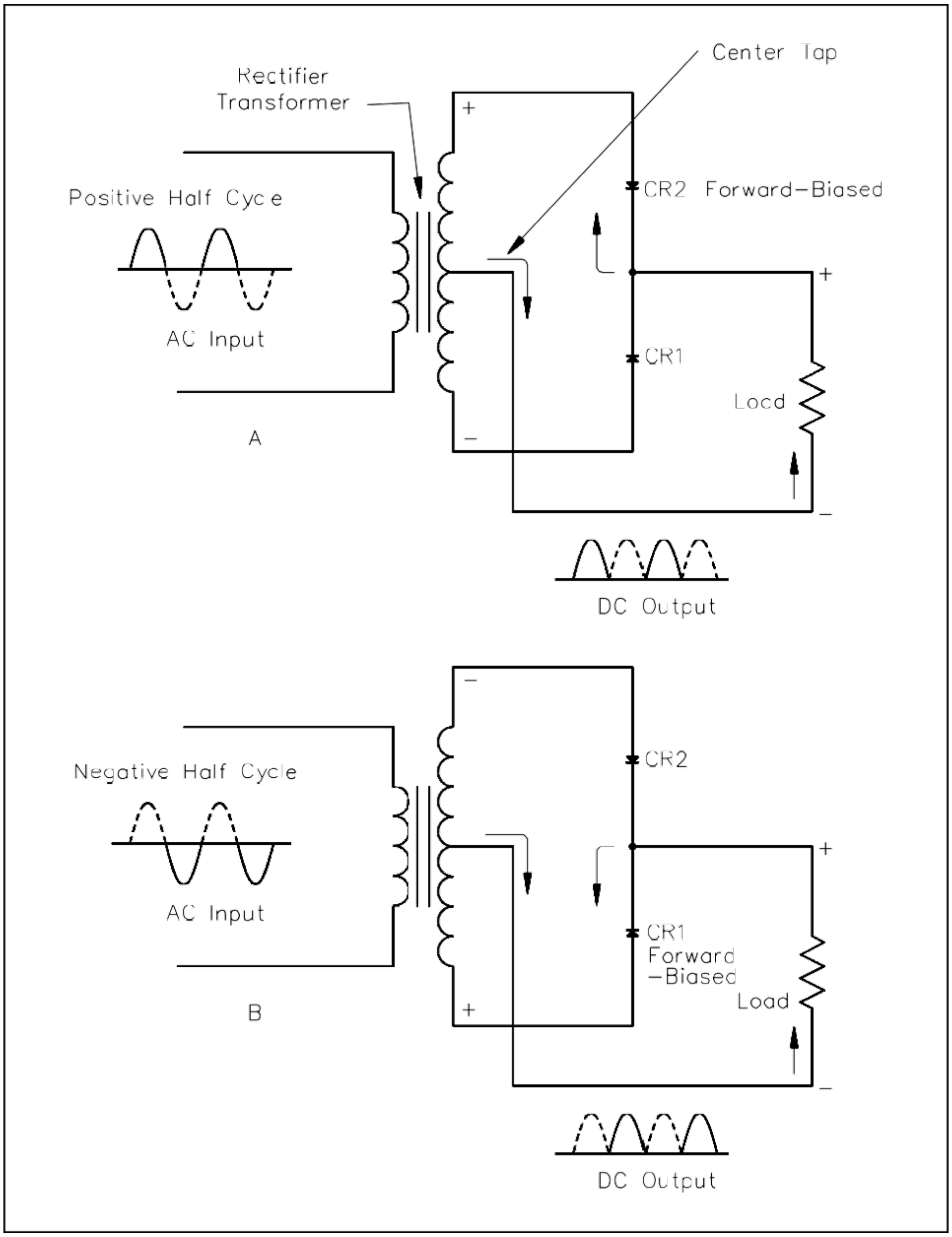

Figure 7 Full-Wave Rectifier

Another type of full-wave rectifier circuit is the full-wave bridge rectifier. This circuit utilizes four diodes. These diodes' actions during each half cycle of the applied AC input voltage are shown in Figure 8. The output of this circuit then becomes a pulsating DC, with all of the waves of the input AC being transferred. The output looks identical to that obtained from a full-wave rectifier (Figure 7).

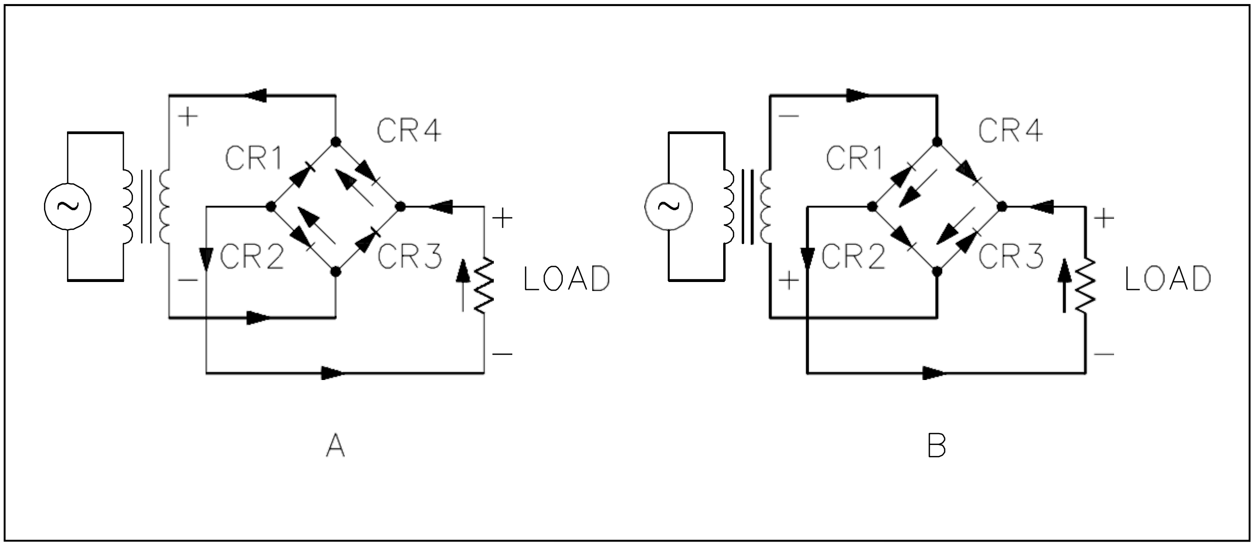

Figure 8 Bridge Rectifier Circuit

Summary

The important information concerning DC sources is summarized below.

DC Sources Summary

- There are four common ways that DC voltages are produced.

  - Batteries
  - DC Generators
  - Thermocouples
  - Rectifiers

- Thermocouples convert energy from temperature into a DC voltage. This voltage can be used to measure temperature.

- A rectifier converts AC to DC.

- There are two types of rectifiers.

  - Half-Wave rectifiers
  - Full-Wave rectifiers

- Half-wave rectifiers convert the AC to a pulsating DC and convert only one-half of the sine wave.

- Full-wave rectifiers convert the AC to a pulsating DC and convert all of the sine wave.

DC CIRCUIT TERMINOLOGY

Before operations with DC circuits can be studied, an understanding of the types of circuits and common circuit terminology associated with circuits is essential.

EO 1.4 **Given a diagram, IDENTIFY it as one of the following types:**
- **a. Schematic diagram**
- **b. One-line diagram**
- **c. Block diagram**
- **d. Wiring diagram**

EO 1.5 **DEFINE the following terms:**
- **a. Resistivity**
- **b. Temperature coefficient of resistance**
- **c. Closed circuit**
- **d. Open circuit**
- **e. Short circuit**
- **f. Series circuit**
- **g. Parallel circuit**
- **h. Equivalent resistance**

EO 1.6 **Given a circuit, DETERMINE whether the circuit is an open circuit or a closed circuit.**

Schematic Diagram

Schematic diagrams are the standard means by which we communicate information in electrical and electronics circuits. On schematic diagrams, the component parts are represented by graphic symbols, some of which were presented earlier in Module 1. Because graphic symbols are small, it is possible to have diagrams in a compact form. The symbols and associated lines show how circuit components are connected and the relationship of those components with one another.

As an example, let us look at a schematic diagram of a two-transistor radio circuit (Figure 9). This diagram, from left to right, shows the components in the order they are used to convert radio waves into sound energy. By using this diagram it is possible to trace the operation of the circuit from beginning to end. Due to this important feature of schematic diagrams, they are widely used in construction, maintenance, and servicing of all types of electronic circuits.

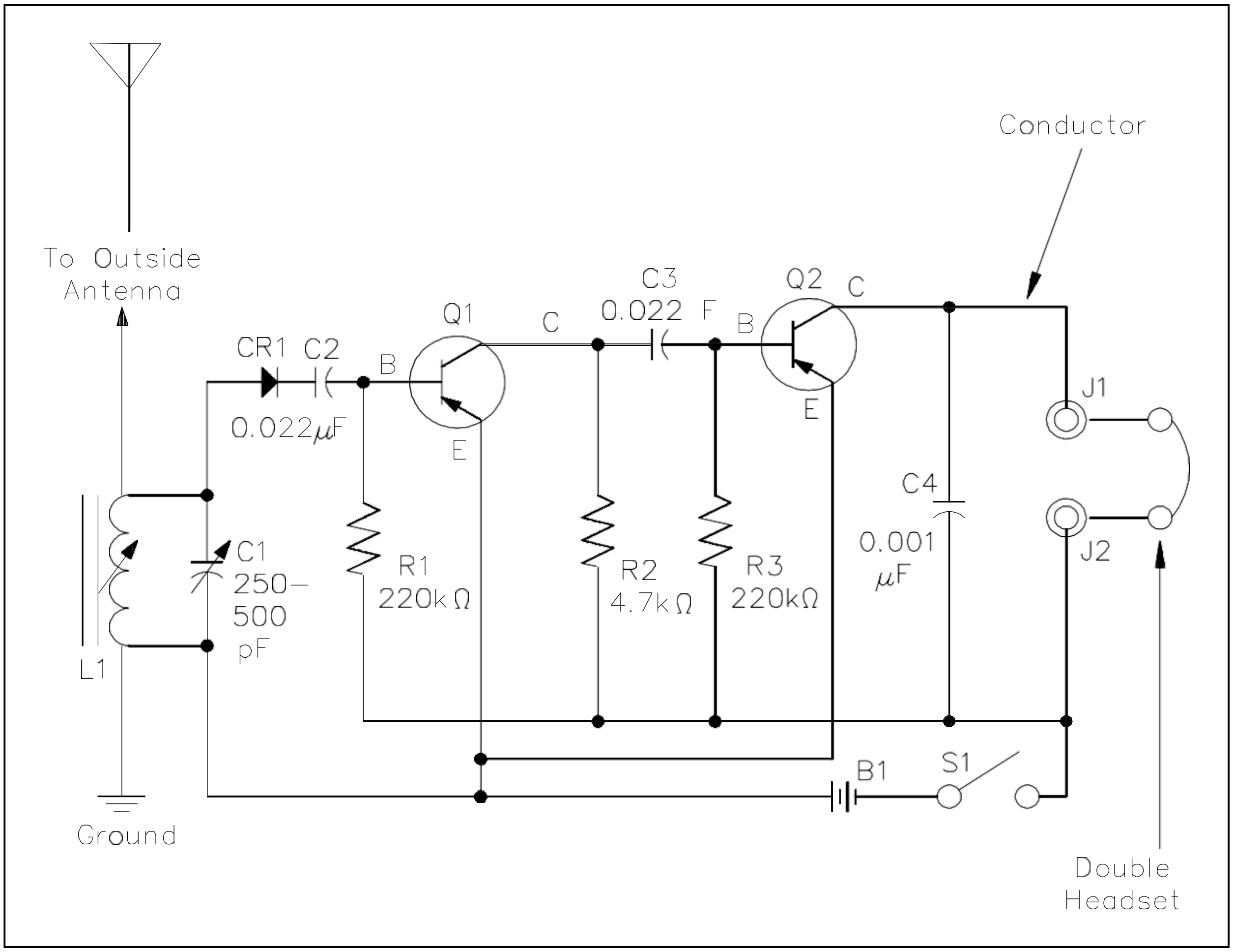

Figure 9 Schematic Diagram

One-Line Diagram

The one-line, or single-line, diagram shows the components of a circuit by means of single lines and the appropriate graphic symbols. One-line diagrams show two or more conductors that are connected between components in the actual circuit. The one-line diagram shows all pertinent information about the sequence of the circuit, but does not give as much detail as a schematic diagram. Normally, the one-line diagram is used to show highly complex systems without showing the actual physical connections between components and individual conductors.

As an example, Figure 10 shows a typical one-line diagram of an electrical substation.

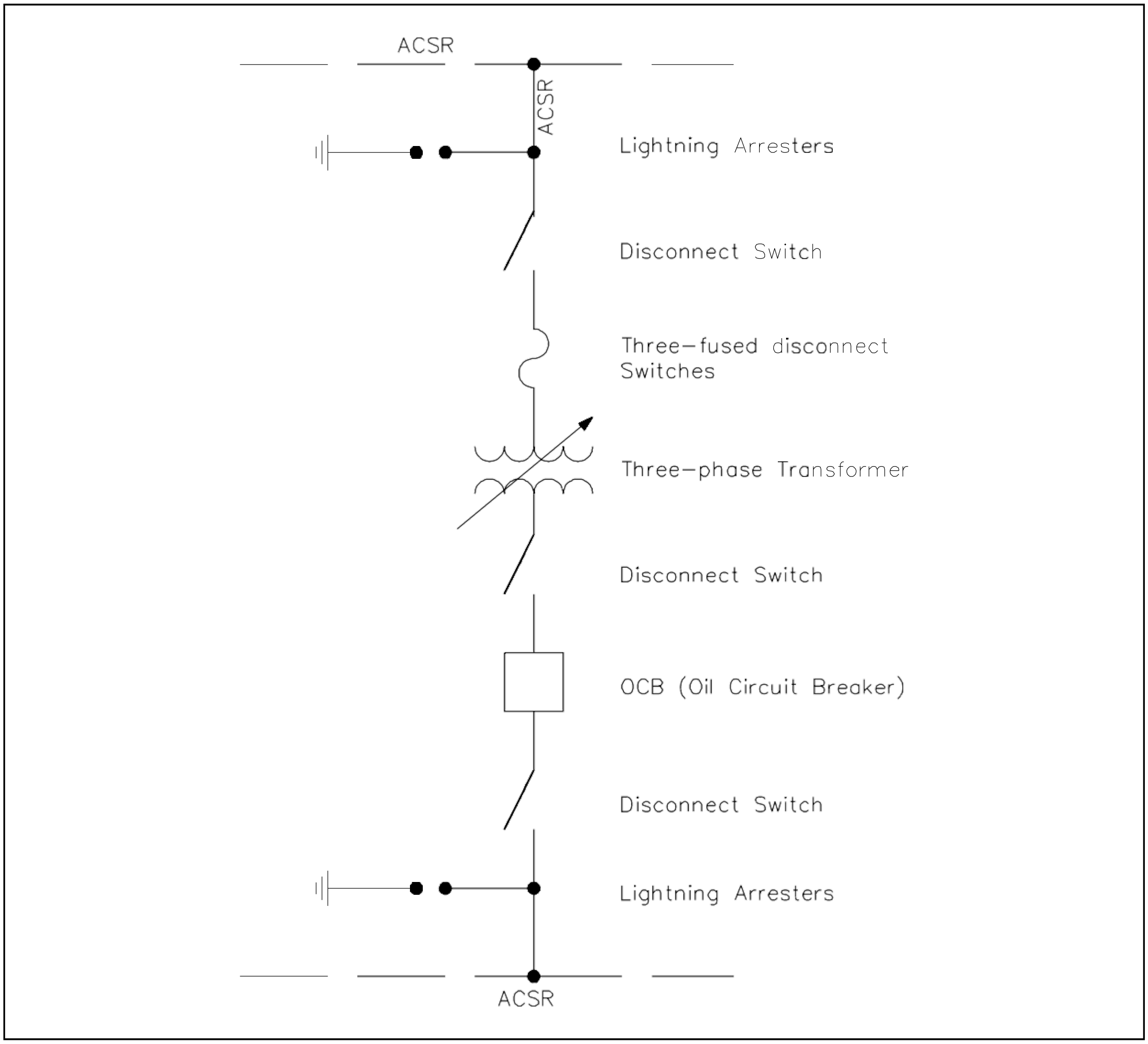

Figure 10 One-Line Diagram

Block Diagram

A block diagram is used to show the relationship between component groups, or stages in a circuit. In block form, it shows the path through a circuit from input to output (Figure 11). The blocks are drawn in the form of squares or rectangles connected by single lines with arrowheads at the terminal end, showing the direction of the signal path from input to output. Normally, the necessary information to describe the stages of components is contained in the blocks.

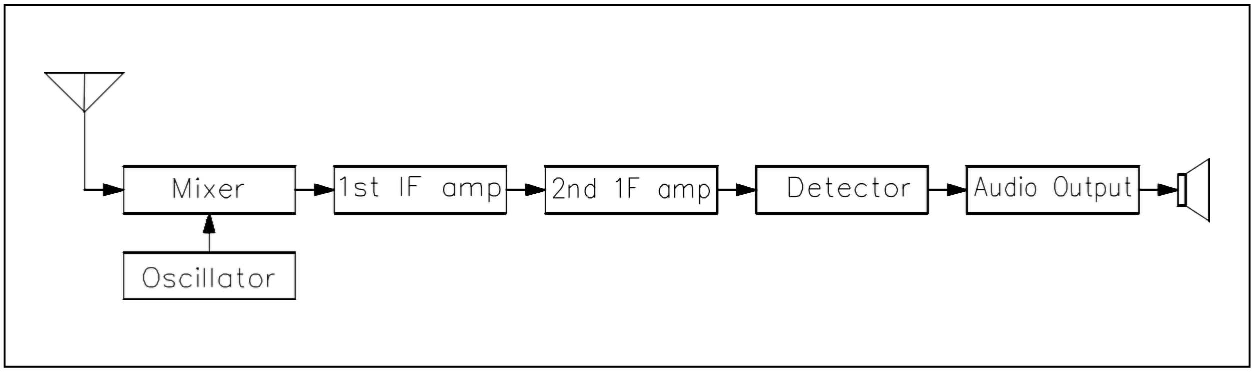

Figure 11 Block Diagram

Wiring Diagram

A wiring diagram is a very simple way to show wiring connections in an easy-to-follow manner. These types of diagrams are normally found with home appliances and automobile electrical systems (Figure 12). Wiring diagrams show the component parts in pictorial form, and the components are identified by name. Most wiring diagrams also show the relative location of component parts and color coding of conductors or leads.

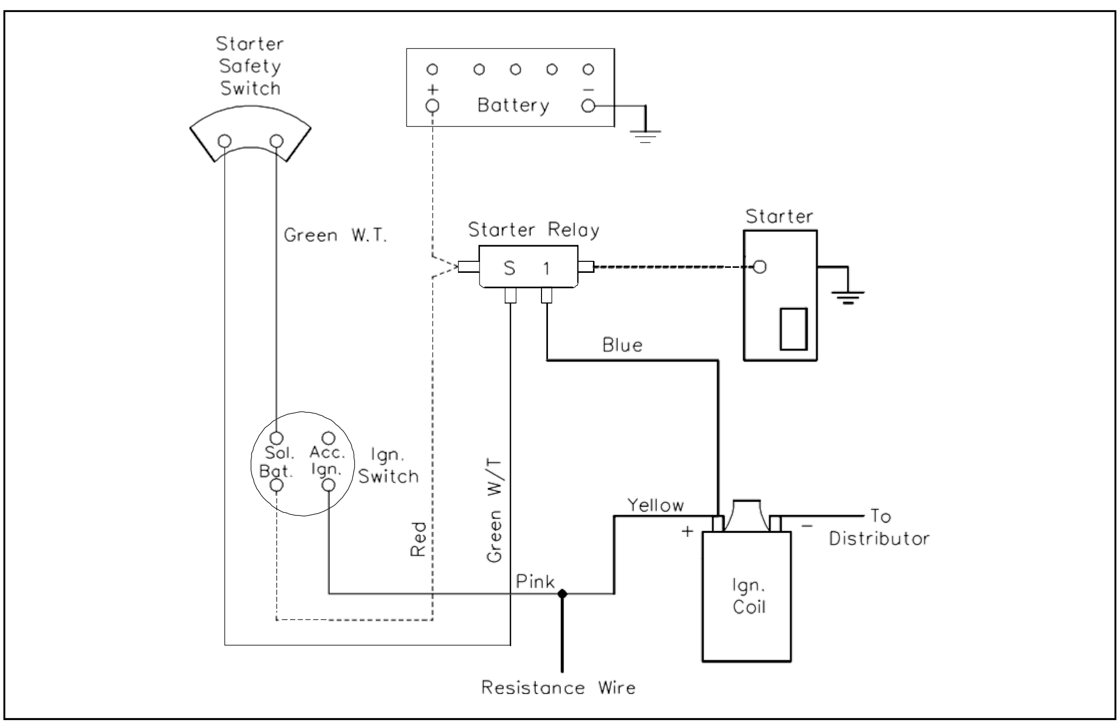

Figure 12 Wiring Diagram

Resistivity

Resistivity is defined as the measure of the resistance a material imposes on current flow. The resistance of a given length of conductor depends upon the resistivity of that material, the length of the conductor, and the cross-sectional area of the conductor, according to Equation (2-1).

$$R = \rho \frac{L}{A} \qquad (2\text{-}1)$$

where

R = resistance of conductor, Ω

ρ = specific resistance or resistivity Ω-cm

L = length of conductor, cm

A = cross-sectional area of conductor, cm^2

The resistivity ρ (rho) allows different materials to be compared for resistance, according to their nature, without regard to length or area. The higher the value of ρ, the higher the resistance.

Table 1 gives resistivity values for metals having the standard wire size of one foot in length and a cross-sectional area of 1 cm^2.

TABLE 1
Properties of Conducting Materials

Material	ρ = Resistivity at 20°C-cm-Ω/ft (a)
Aluminum	17
Carbon	(b)
Constantan	295
Copper	10.4
Gold	14
Iron	58
Nichrome	676
Nickel	52
Silver	9.8
Tungsten	33.8

(a) Precise values depend on exact composition of material.
(b) Carbon has 2500-7500 times the resistance of copper.

Temperature Coefficient of Resistance

Temperature coefficient of resistance, α (alpha), is defined as the amount of change of the resistance of a material for a given change in temperature. A positive value of α indicates that R increases with temperature; a negative value of α indicates R decreases; and zero α indicates that R is constant. Typical values are listed in Table 2.

TABLE 2
Temperature Coefficients for Various Materials

Material	Temperature Coefficient, Ω per °C
Aluminum	0.004
Carbon	-0.0003
Constantan	0 (avg)
Copper	0.004
Gold	0.004
Iron	0.006
Nichrome	0.0002
Nickel	0.005

For a given material, α may vary with temperature; therefore, charts are often used to describe how resistance of a material varies with temperature.

An increase in resistance can be approximated from equation (2-2).

$$R_t = R_o + R_o(\alpha\Delta T) \tag{2-2}$$

where

R_t = higher resistance at higher temperatures

R_o = resistance at 20°C

α = temperature coefficient

ΔT = temperature rise above 20°C

Electric Circuit

Each electrical circuit has at least four basic parts: (1) a source of electromotive force, (2) conductors, (3) load or loads, and (4) some means of control. In Figure 13, the source of EMF is the battery; the conductors are wires which connect the various component parts; the resistor is the load; and a switch is used as the circuit control device.

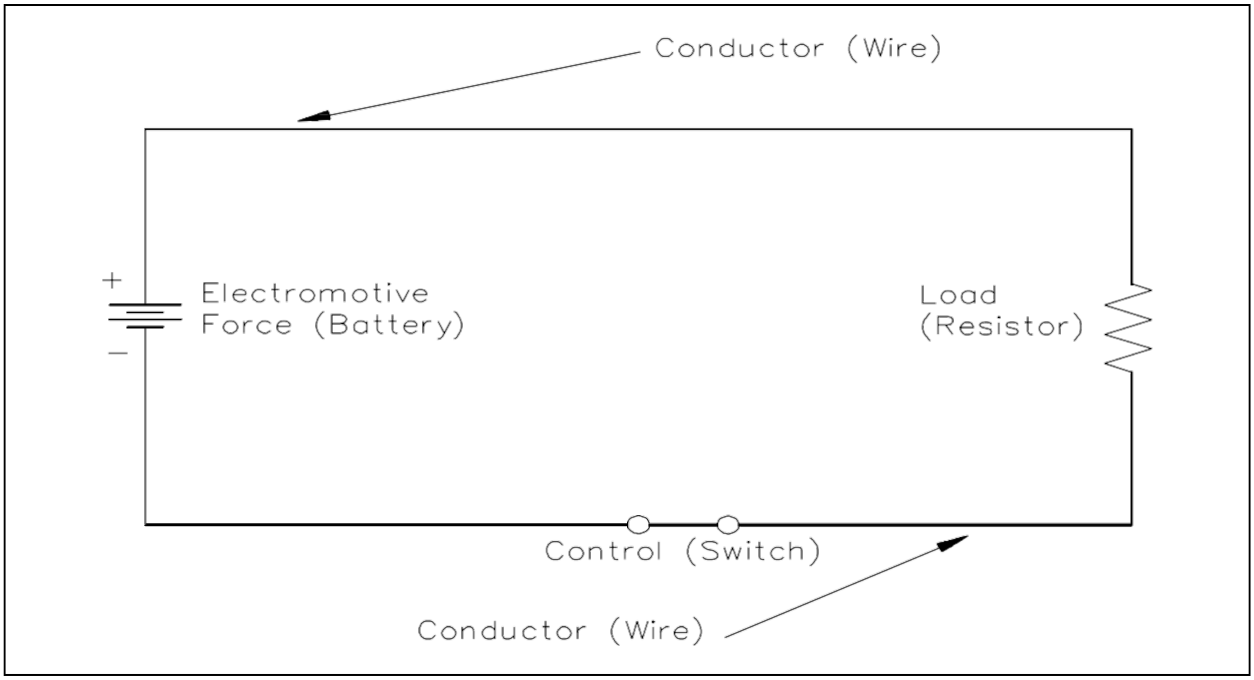

Figure 13 Closed Circuit

A *closed circuit* (Figure 13) is an uninterrupted, or unbroken, path for current from the source (EMF), through the load, and back to the source.

An *open circuit*, or incomplete circuit, (Figure 14) exists if a break in the circuit occurs; this prevents a complete path for current flow.

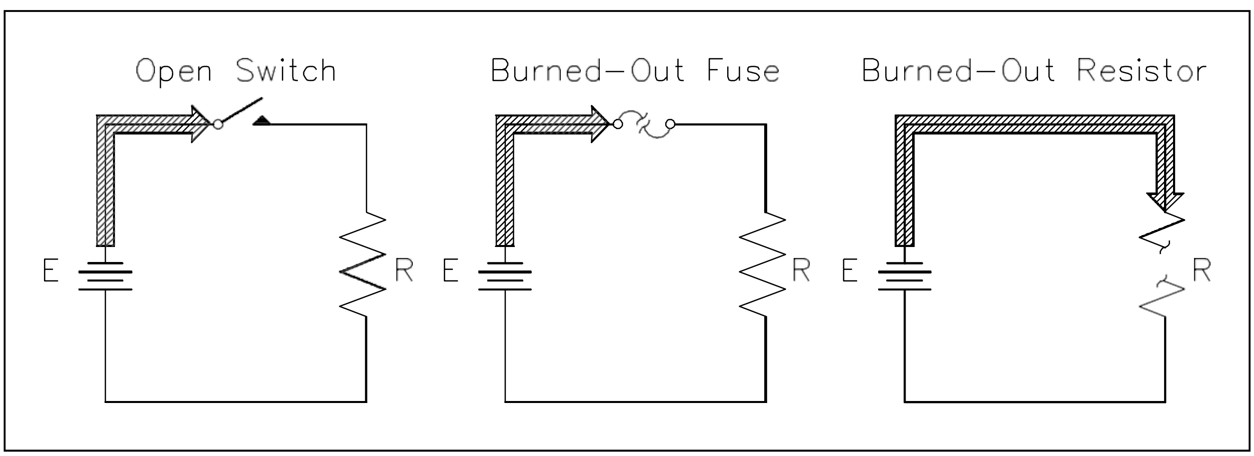

Figure 14 Open Circuit

A *short circuit* is a circuit which offers very little resistance to current flow and can cause dangerously high current flow through a circuit (Figure 15). Short circuits are usually caused by an inadvertent connection between two points in a circuit which offers little or no resistance to current flow. Shorting resistor R in Figure 15 will probably cause the fuse to blow.

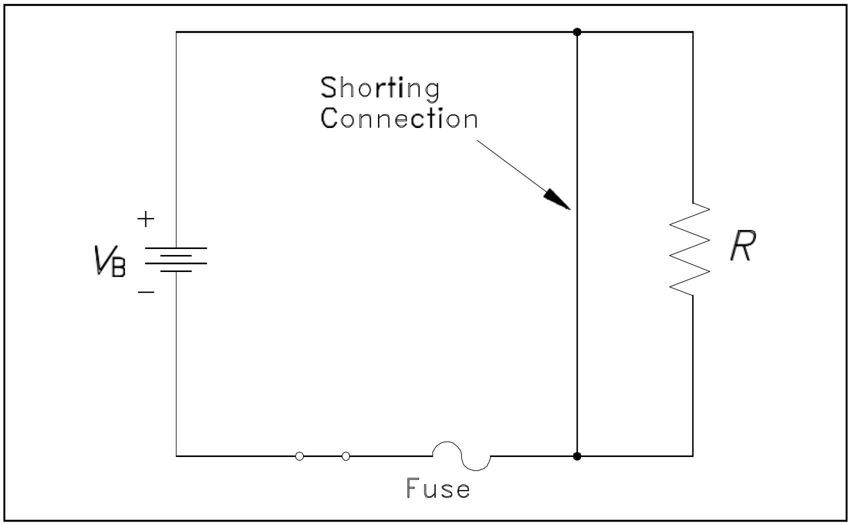

Figure 15 Short Circuit

Series Circuit

A *series circuit* is a circuit where there is only one path for current flow. In a series circuit (Figure 16), the current will be the same throughout the circuit. This means that the current flow through R_1 is the same as the current flow through R_2 and R_3.

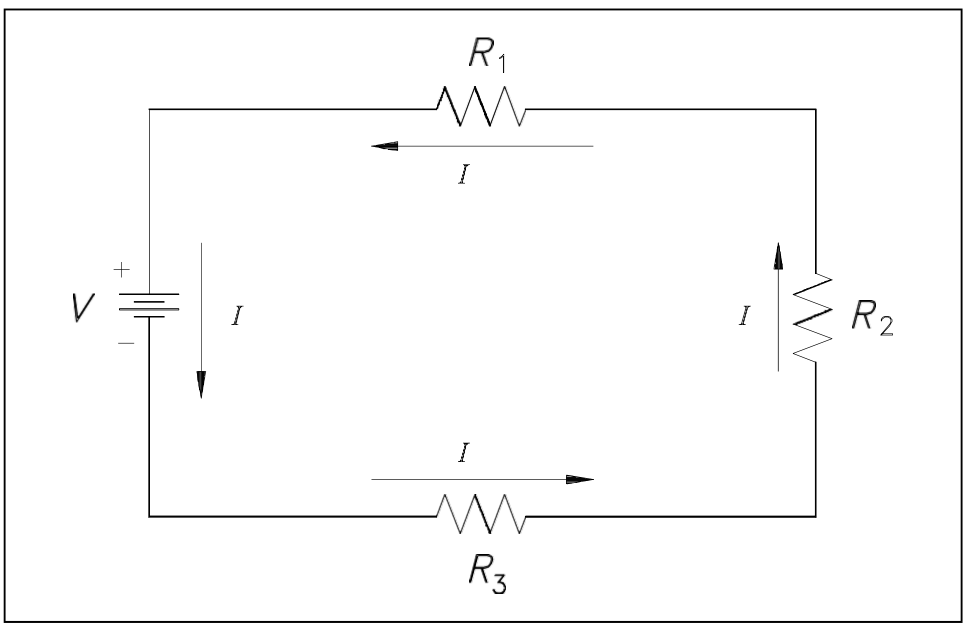

Figure 16 Series Circuit

Parallel Circuit

Parallel circuits are those circuits which have two or more components connected across the same voltage source (Figure 17). Resistors R_1, R_2, and R_3 are in parallel with each other and the source. Each parallel path is a branch with its own individual current. When the current leaves the source V, part I_1 of I_T will flow through R_1; part I_2 will flow through R_2; and part I_3 will flow through R_3. Current through each branch can be different; however, voltage throughout the circuit will be equal.

$$V = V_1 = V_2 = V_3.$$

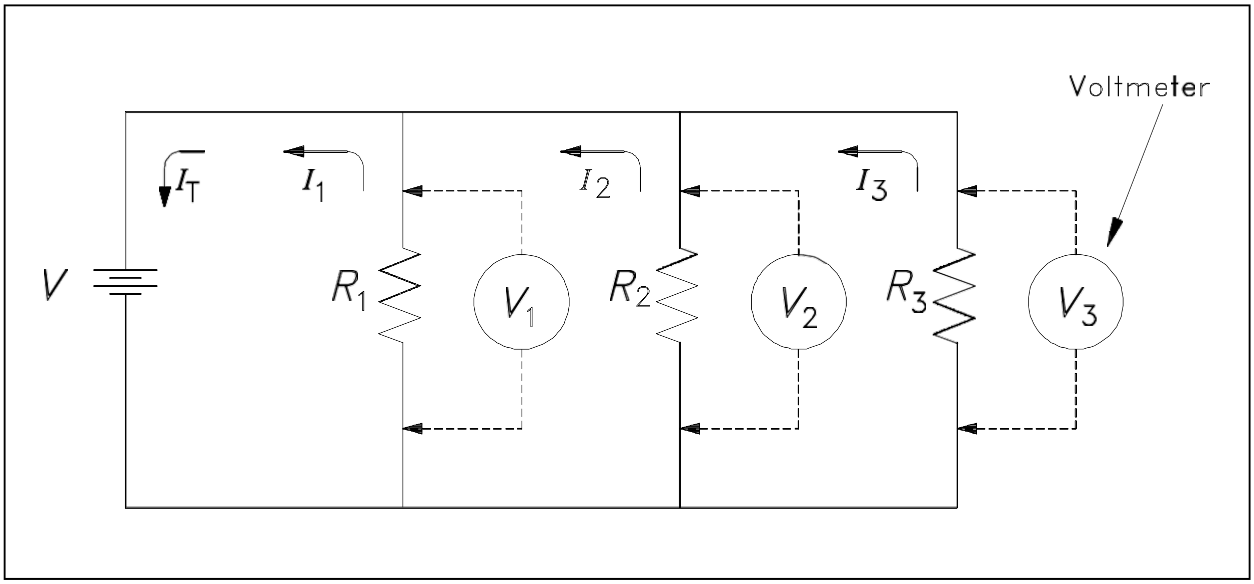

Figure 17 Parallel Circuit

Equivalent Resistance

In a parallel circuit, the total resistance of the resistors in parallel is referred to as *equivalent resistance*. This can be described as the total circuit resistance as seen by the voltage source. In all cases, the equivalent resistance will be less than any of the individual parallel circuit resistors. Using Ohm's Law, equivalent resistance (R_{EQ}) can be found by dividing the source voltage (V) by the total circuit current (I_T), as shown in Figure 17.

$$R_{EQ} = \frac{V}{I_t}$$

Summary

The important information concerning basic DC circuits is summarized below.

DC Circuit Terminology Summary

- There are four types of circuit diagrams.

  - Schematic diagram
  - One-line diagram
  - Block diagram
  - Wiring diagram

- Resistivity is defined as the measure of the resistance a material imposes on current flow.

- Temperature coefficient of resistance, α (alpha), is defined as the amount of change of the resistance of a material for a given change in temperature.

- A closed circuit is one that has a complete path for current flow.

- An open circuit is one that does not have a complete path for current flow.

- A short circuit is a circuit with a path that has little or no resistance to current flow.

- A series circuit is one where there is only one path for current flow.

- A parallel circuit is one which has two or more components connected across the same voltage source.

- Equivalent resistance is the total resistance of the resistors in parallel.

BASIC DC CIRCUIT CALCULATIONS

Each type of DC circuit contains certain characteristics that determine the way its voltage and current behave. To begin analysis of the voltages and currents at each part of a circuit, an understanding of these characteristics is necessary.

EO 1.7 **Given a circuit, CALCULATE total resistance for a series or parallel circuit.**

EO 1.8 **DESCRIBE what is meant by the term "voltage divider."**

EO 1.9 **DESCRIBE what is meant by the term "current division."**

Series Resistance

The total resistance in a series circuit is equal to the sum of all the parts of that circuit, as shown in equation (2-3).

$$R_T = R_1 + R_2 + R_3 \ldots \text{etc.} \qquad (2\text{-}3)$$

where

R_T = resistance total

R_1, R_2, and R_3 = resistance in series

Example: A series circuit has a 60Ω, a 100Ω, and a 150Ω resistor in series (Figure 18). What is the total resistance of the circuit?

Solution:

$$\begin{aligned} R_T &= R_1 + R_2 + R_3 \\ &= 60 + 100 + 150 \\ &= 310\ \Omega \end{aligned}$$

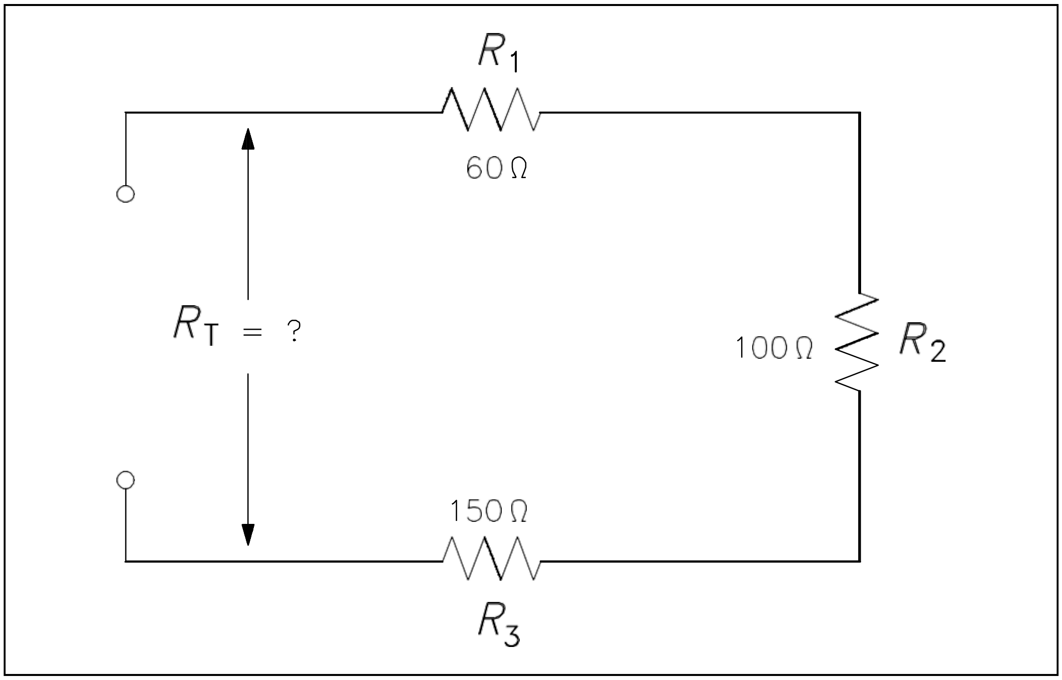

Figure 18 Resistance in a Series Circuit

The total voltage across a series circuit is equal to the sum of the voltages across each resistor in the circuit (Figure 19) as shown in equation (2-4).

$$V_T = V_1 + V_2 + V_3 \text{ ... etc.} \tag{2-4}$$

where

V_T = total voltage

V_1 = voltage across R_1

V_2 = voltage across R_2

V_3 = voltage across R_3

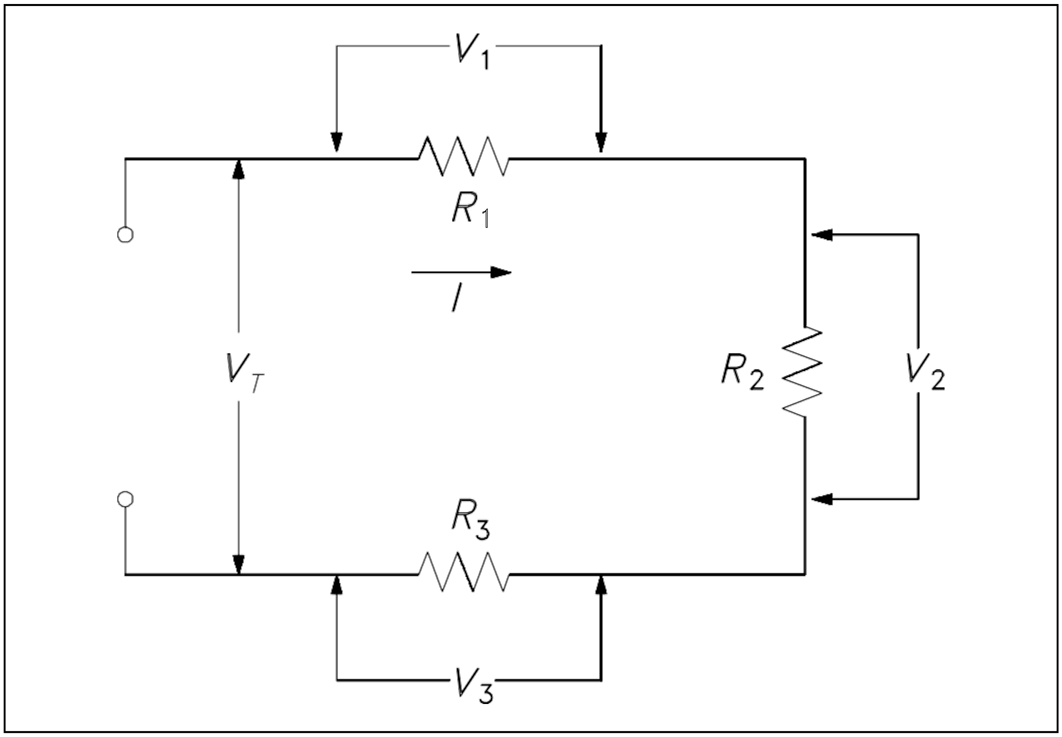

Figure 19 Voltage Drops in a Series Circuit

Ohm's law may now be applied to the entire series circuit or to individual component parts of the circuit. When used on individual component parts, the voltage across that part is equal to the current times the resistance of that part. For the circuit shown in Figure 20, the voltage can be determined as shown below.

$V_1 = IR_1$

$V_2 = IR_2$

$V_3 = IR_3$

$V_T = V_1 + V_2 + V_3$

$V_T = 10 \text{ volts} + 24 \text{ volts} + 36 \text{ volts}$

$V_T = 70 \text{ volts}$

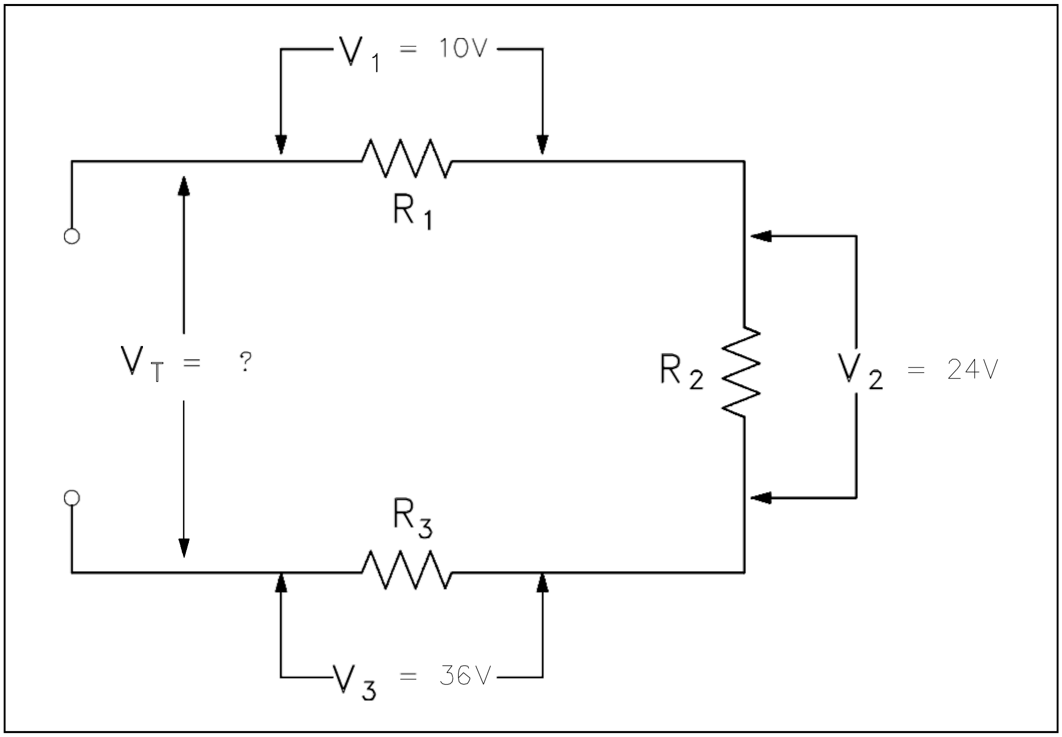

Figure 20 Voltage Total in a Series Circuit

To find the total voltage across a series circuit, multiply the current by the total resistance as shown in equation (2-5).

$$V_T = IR_T \tag{2-5}$$

where

V_T	=	total voltage
I	=	current
R_T	=	total resistance

Example 1: A series circuit has a 50Ω, a 75Ω, and a 100Ω resistor in series (Figure 21). Find the voltage necessary to produce a current of 0.5 amps.

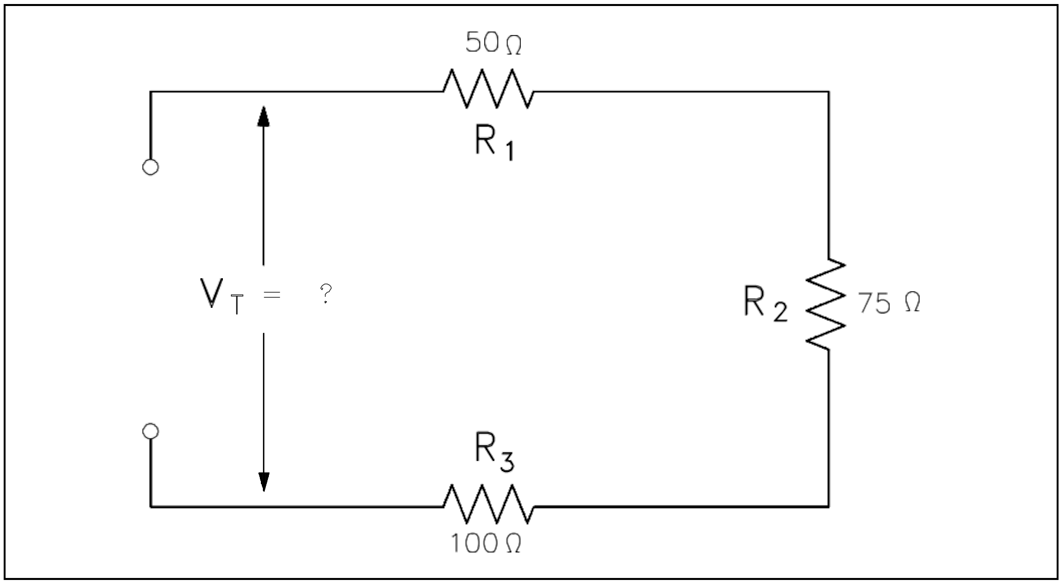

Figure 21 Example 1 Series Circuit

Solution:

Step 1: Find circuit current. As we already know, current is the same throughout a series circuit, which is already given as 0.5 amps.

Step 2: Find R_T.

$$R_T = R_1 + R_2 + R_3$$

$$R_T = 50\ \Omega + 75\ \Omega + 100\ \Omega$$

$$R_T = 225\ \Omega$$

Step 3: Find V_T. Use Ohm's law.

$$V_T = IR_T$$

$$V_T = (0.5\text{ amps})(225\ \Omega)$$

$$V_T = 112.5\text{ volts}$$

Example 2: A 120 V battery is connected in series with three resistors: 40Ω, 60Ω, and 100Ω (Figure 22). Find the voltage across each resistor.

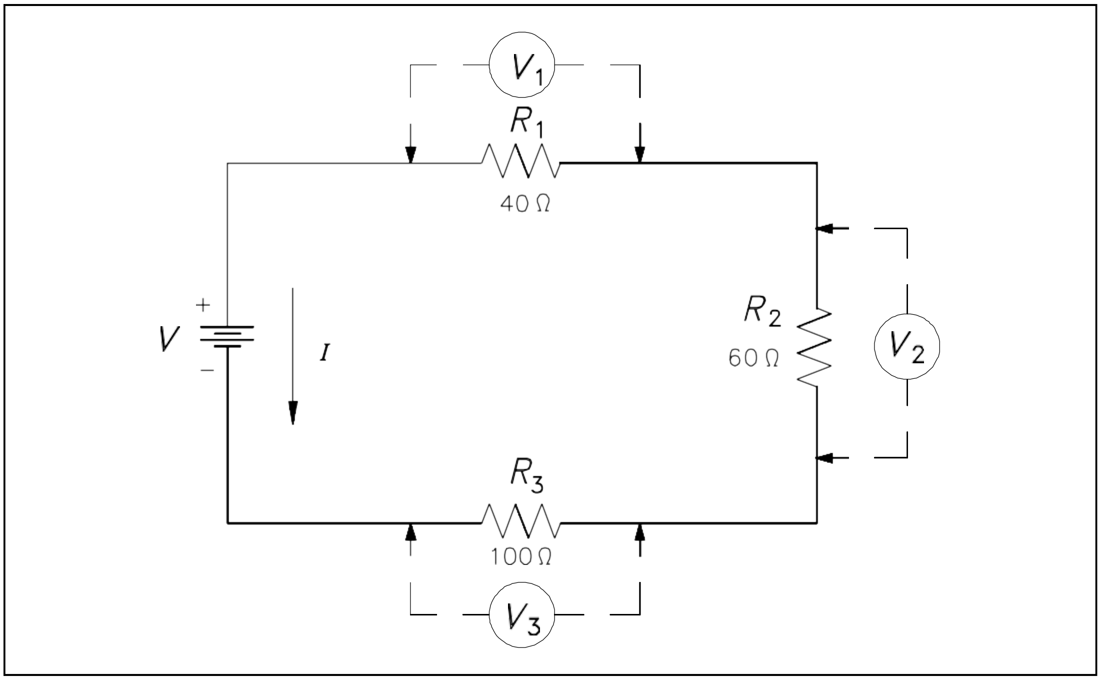

Figure 22 Example 2 Series Circuit

Solution:

Step 1: Find total resistance.

$R_T = R_1 + R_2 + R_3$

$R_T = 40\ \Omega + 60\ \Omega + 100\ \Omega$

$R_T = 200$ ohms

Step 2: Find circuit current (I).

$V_T = IR_T$

Solving for I:

$$I = \frac{V_T}{R_T}$$

$$I = \frac{120 \text{ volts}}{200\ \Omega}$$

$$I = 0.6 \text{ amps}$$

Step 3: Find the voltage across each component.

$V_1 = IR_1$
$V_1 = (0.6 \text{ amps})(40\ \Omega)$
$V_1 = 24 \text{ volts}$

$V_2 = IR_2$
$V_2 = (0.6 \text{ amps})(60\ \Omega)$
$V_2 = 36 \text{ volts}$

$V_3 = IR_3$
$V_3 = (0.6 \text{ amps})(100\ \Omega)$
$V_3 = 60 \text{ volts}$

The voltages of V_1, V_2, and V_3 in Example 2 are known as "voltage drops" or "IR drops." Their effect is to reduce the available voltage to be applied across the other circuit components. The sum of the voltage drops in any series circuit is always equal to the applied voltage. We can verify our answer in Example 2 by using equation (2-4).

$$V_T = V_1 + V_2 + V_3$$
$$120 \text{ volts} = 24 \text{ volts} + 36 \text{ volts} + 60 \text{ volts}$$
$$120 \text{ volts} = 120 \text{ volts}$$

Parallel Currents

The sum of the currents flowing through each branch of a parallel circuit is equal to the total current flow in the circuit. Using Ohm's Law, the branch current for a three branch circuit equals the applied voltage divided by the resistance as shown in equations (2-6), (2-7), and (2-8).

Branch 1: $$I_1 = \frac{V_1}{R_1} = \frac{V}{R_1} \qquad (2\text{-}6)$$

Branch 2: $$I_2 = \frac{V_2}{R_2} = \frac{V}{R_2} \qquad (2\text{-}7)$$

Branch 3: $$I_3 = \frac{V_2}{R_2} = \frac{V}{R_2} \qquad (2\text{-}8)$$

Example 1: Two resistors, each drawing 3A, and a third resistor, drawing 2A, are connected in parallel across a 115 volt source (Figure 23). What is total current?

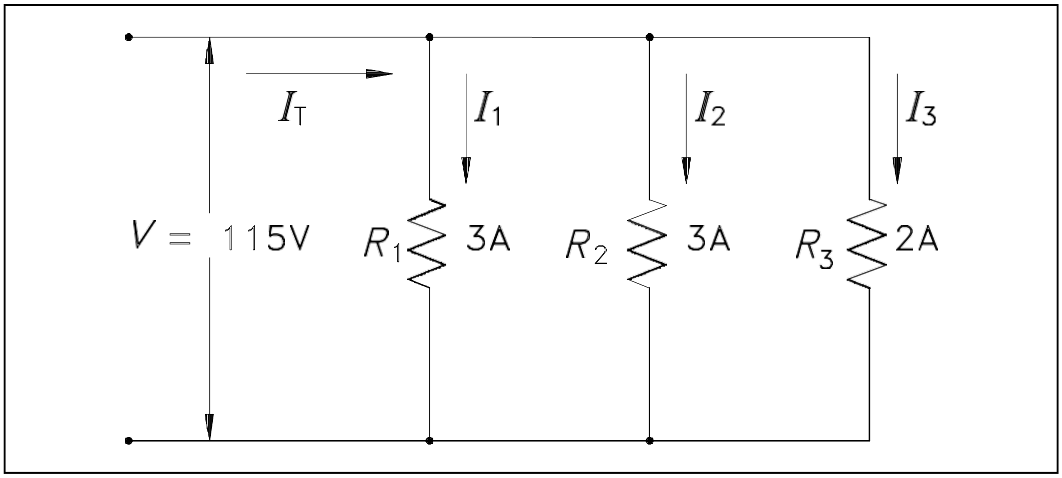

Figure 23 Example 1 Parallel Circuit

$I_T = I_1 + I_2 + I_3$
$I_T = 3A + 3A + 2A$
$I_T = 8A$

Example 2: Two branches, R_1 and R_2, are across a 120 V power source. The total current flow is 30 A (Figure 24). Branch R_1 takes 22 amps. What is the current flow in Branch R_2?

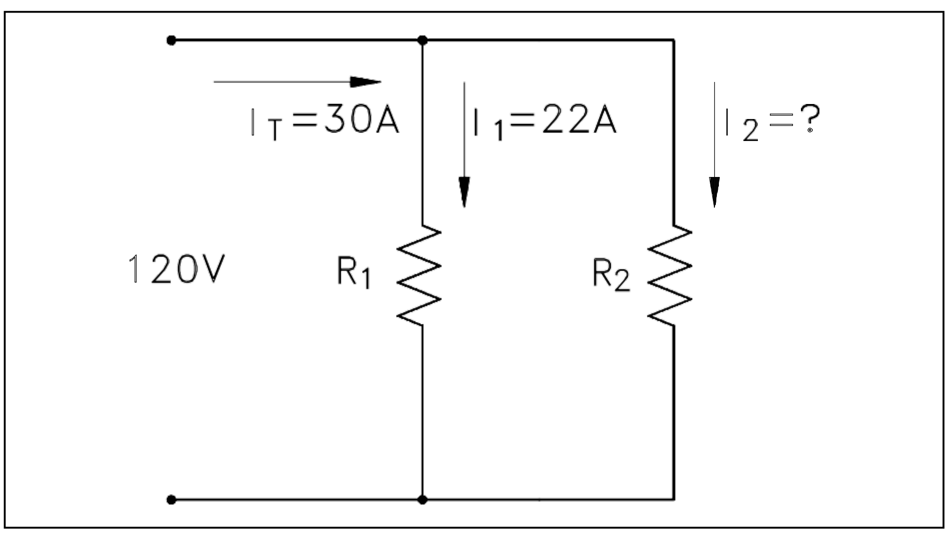

Figure 24 Example 2 Parallel Circuit

$I_T = I_1 + I_2$
$I_2 = I_T - I_1$
$I_2 = 30 - 22$
$I_2 = 8$ amps

Example 3: A parallel circuit consists of $R_1 = 15\Omega$, $R_2 = 20\Omega$ and $R_3 = 10\Omega$, with an applied voltage of 120 V (Figure 25). What current will flow through each branch?

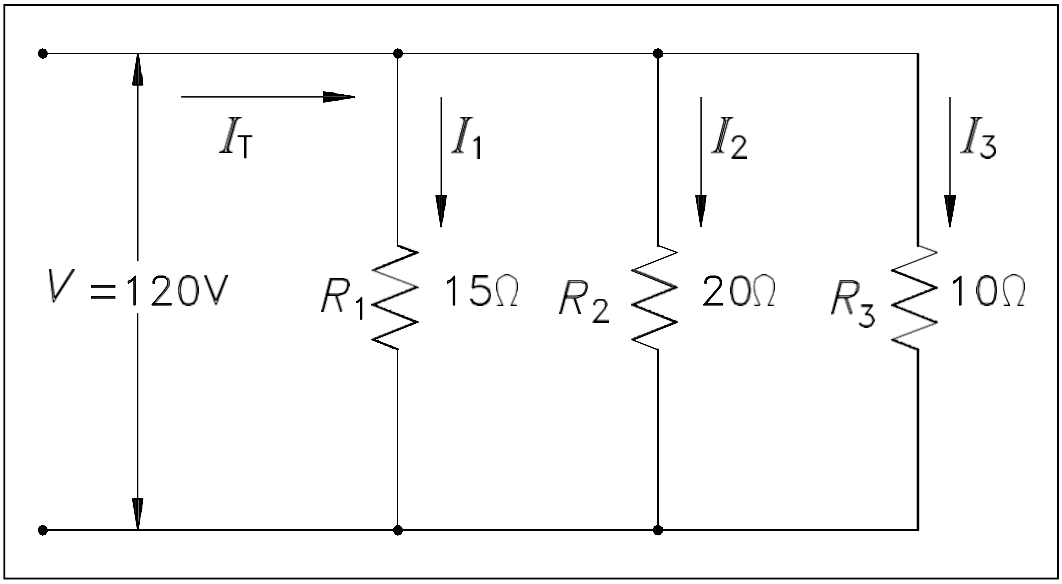

Figure 25 Example 3 Parallel Circuit

$$I_1 = \frac{V}{R_1} = \frac{120}{15} = 8A$$

$$I_2 = \frac{V}{R_2} = \frac{120}{20} = 6A$$

$$I_3 = \frac{V}{R_3} = \frac{120}{10} = 12A$$

$$I_T = I_1 + I_2 + I_3$$

$$I_T = 8A + 6A + 12A$$

$$I_T = 26A$$

Resistance in Parallel

Total resistance in a parallel circuit can be found by applying Ohm's Law. Divide the voltage across the parallel resistance by the total line current as shown in equation (2-9).

$$R_T = \frac{V}{I_T} \quad (2\text{-}9)$$

Example: Find the total resistance of the circuit shown in Figure 25 if the line voltage is 120 V and total current is 26A.

$$R_T = \frac{V}{I_T} = \frac{120}{26} = 4.62\ \Omega$$

The total load connected to a 120 V source is the same as the single "equivalent resistance" of 4.62Ω connected across the source (Figure 26). Equivalent resistance is the total resistance a combination of loads present to a circuit.

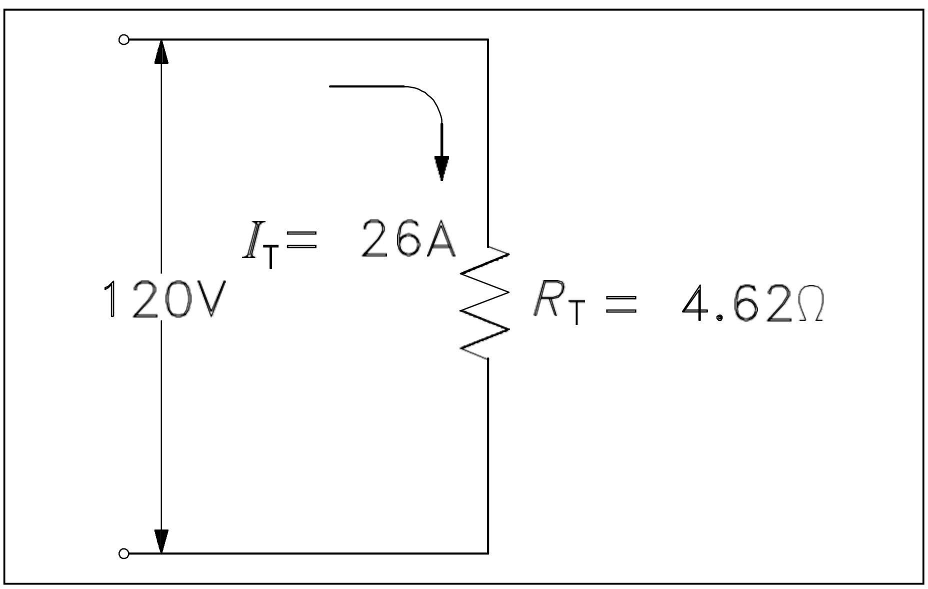

Figure 26 Equivalent Resistance in a Parallel Circuit

The total resistance in a parallel circuit can also be found by using the equation (2-10).

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \frac{1}{R_N} \tag{2-10}$$

Example 1: Find the total resistance of a 4Ω, an 8Ω, anda 16Ω resistor in parallel (Figure 27).

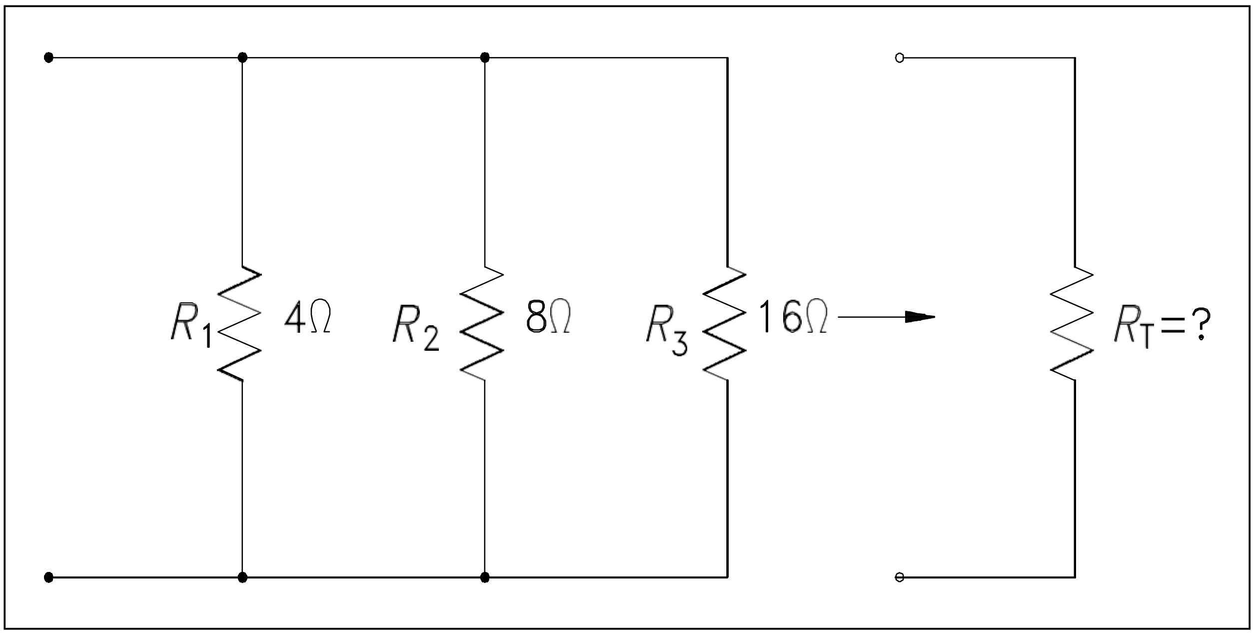

Figure 27 Total Resistance in a Parallel Circuit

Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$$

$$\frac{1}{R_T} = \frac{4}{16} + \frac{2}{16} + \frac{1}{16} = \frac{7}{16}$$

$$R_T = \frac{16}{7} = 2.29\ \Omega$$

Note: Whenever resistors are in parallel, the total resistance is always smaller than any single branch.

Example 2: Now add a fourth resistance of 4Ω in parallel to the circuit in Figure 27. What is the new total resistance of the circuit?

Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_T} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{4}$$

$$\frac{1}{R_T} = \frac{4}{16} + \frac{2}{16} + \frac{1}{16} + \frac{4}{16} = \frac{11}{16}$$

$$R_T = \frac{16}{11} = 1.4\ \Omega$$

Simplified Formulas

Total resistance of equal resistors in a parallel circuit is equal to the resistance of one resistor divided by the number of resistors.

$$R_T = \frac{R}{N}$$

where

R_T = total resistance

R = resistance of one resistor

N = number of resistors

Example: Five lamps, each with a resistance of 40Ω, are connected in parallel. Find total resistance.

$$R = R_1 = R_2 = R_3 = R_4 = R_5 = 40\Omega$$

$$N = 5$$

$$R_T = \frac{R}{N} = \frac{40}{5} = 8\Omega$$

When any two resistors are unequal in a parallel circuit, it is easier to calculate R_T by multiplying the two resistances and then dividing the product by the sum, as shown in equation (2-11). As shown in equation (2-11), this is valid when there are only two resistors in parallel.

$$R_T = \frac{R_1 R_2}{R_1 + R_2} \tag{2-11}$$

Example: Find the total resistance of a parallel circuit which has one 12Ω and one 4Ω resistor.

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(12)(4)}{12 + 4} = \frac{48}{16} = 3\Omega$$

In certain cases involving two resistors in parallel, it is useful to find an unknown resistor, R_x, to obtain a certain R_T. To find the appropriate formula, we start with equation (2-10) and let the known resistor be R and the unknown resistor be R_x.

$$R_T = \frac{RR_X}{R + R_X}$$

Cross multiply: $R_T R + R_T R_X = RR_X$

Transpose: $RR_X - R_T R_X = R_T R$

Factor: $R_X (R - R_T) = R_T R$

Solve for R_x: $R_X = \frac{R_T R}{R - R_T}$

Example: What value of resistance must be added, in parallel, with an 8Ω resistor to provide a total resistance of 6Ω (Figure 28)?

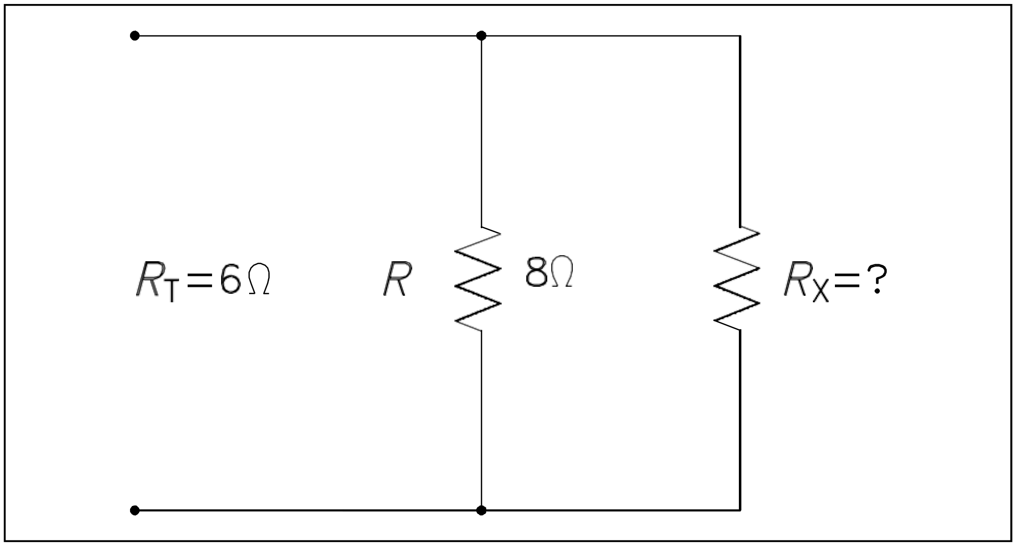

Figure 28 Example Parallel Circuit

Solution:

$$R_X = \frac{RR_T}{R - R_T} = \frac{(8)(6)}{8-6} = \frac{48}{2} = 24\Omega$$

Voltage Divider

A *voltage divider*, or network, is used when it is necessary to obtain different values of voltage from a single energy source. A simple voltage divider is shown in Figure 29. In this circuit, 24 volts is applied to three resistors in series. The total resistance limits the current through the circuit to one ampere. Individual voltages are found as follows using equation (2-12).

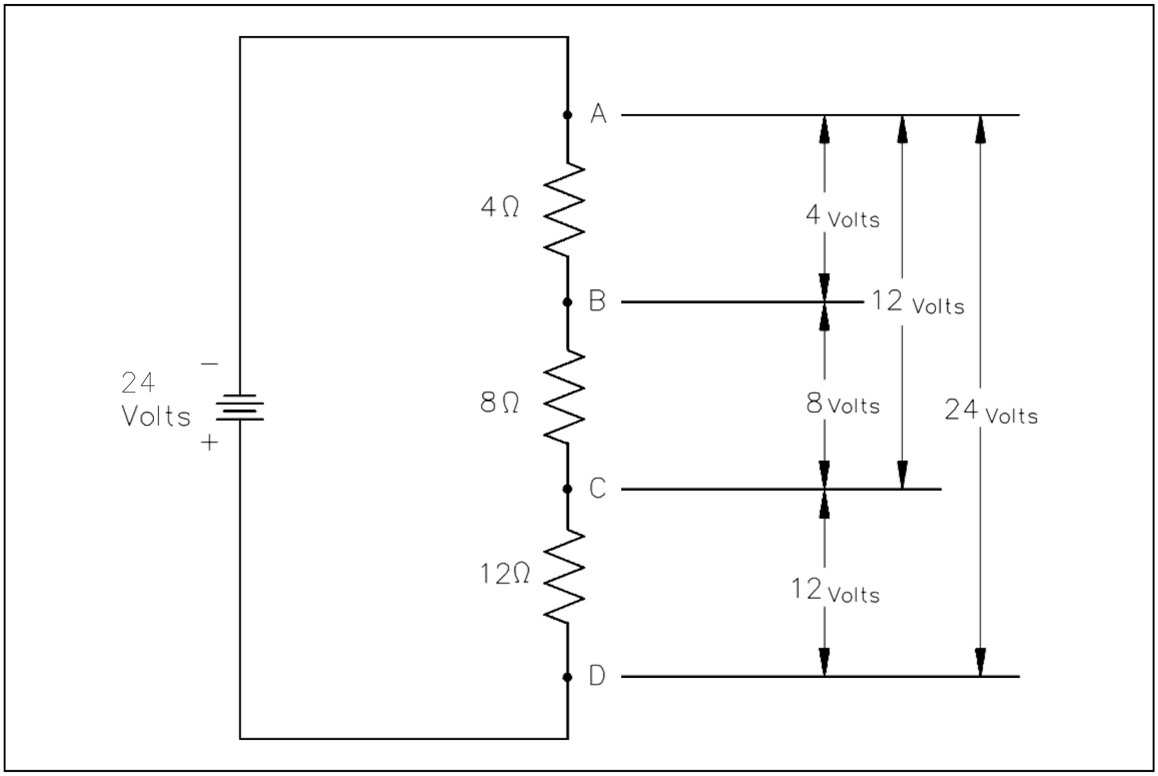

Figure 29 Voltage Divider

$$\text{Total current:}\quad I = \frac{V}{R} = \frac{24}{4 + 8 + 12} = \frac{24}{24} = 1\,\text{amp} \qquad (2\text{-}12)$$

Voltage drop across AB:

$$\begin{aligned} V &= IR \\ &= (1)(4) \\ V &= 4\ \text{Volts} \end{aligned}$$

Voltage drop across BC:

$$\begin{aligned} V &= IR \\ &= (1)(8) \\ V &= 8\ \text{Volts} \end{aligned}$$

Voltage drop across CD:

$$\begin{aligned} V &= IR \\ &= (1)(12) \\ V &= 12\ \text{Volts} \end{aligned}$$

Total voltage drop AC:

$$\begin{aligned} V &= IR \\ &= (1)(8 + 4) = (1)(12) \\ V &= 12\ \text{Volts} \end{aligned}$$

Current Division

Sometimes it is necessary to find the individual branch currents in a parallel circuit when only resistance and total current are known. When only two branches are involved, the current in one branch will be some fraction of I_T. The resistance in each circuit can be used to divide the total current into fractional currents in each branch. This process is known as *current division*.

$$I_1 = \frac{R_2}{R_1 + R_2} I_T$$

$$I_2 = \frac{R_1}{R_1 + R_2} I_T \qquad (2\text{-}13)$$

Note that the equation for each branch current has the opposite R in the numerator. This is because each branch current is inversely proportional to the branch resistance.

Example: Find branch current for I_1 and I_2 for the circuit shown in Figure 30.

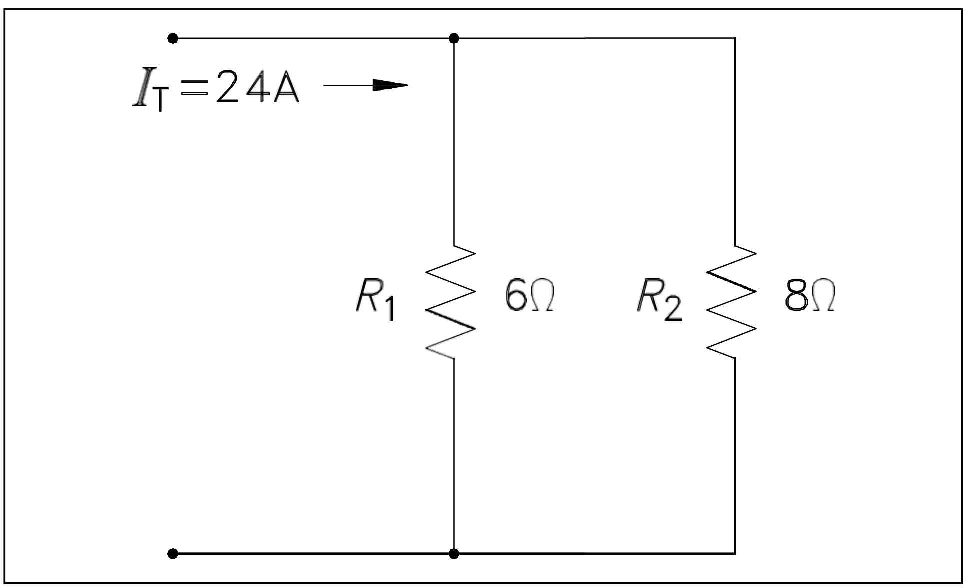

Figure 30 Current Division Example Circuit

Solution:

$$I_1 = \frac{R_2}{R_1 + R_2} I_T = \frac{8}{6+8}(24) = \frac{8}{14}(24) = 13.71 \text{ amps}$$

$$I_2 = \frac{R_1}{R_1 + R_2} I_T = \frac{6}{6+8}(24) = \frac{6}{14}(24) = 10.29 \text{ amps}$$

Since I_1 and I_T were known, we could have also simply subtracted I_1 from I_T to find I_2:

$$\begin{aligned} I_T &= I_1 + I_2 \\ I_2 &= I_T - I_1 \\ &= 24 - 13.71 \\ &= 10.29 \text{ amps} \end{aligned}$$

Summary

The important information in this chapter is summarized below.

Basic DC Circuit Calculations Summary

- Equivalent resistance is a term used to represent the total resistance a combination of loads presents to a circuit.
- A voltage divider is used to obtain different values of voltage from a single energy source.
- Current division is used to determine the current flowing through each leg of a parallel circuit.

VOLTAGE POLARITY AND CURRENT DIRECTION

Before introducing the laws associated with complex DC circuit analysis, the importance of voltage polarity and current direction must be understood. This chapter will introduce the polarities and current direction associated with DC circuits.

EO 1.10 **DESCRIBE the difference between electron flow and conventional current flow.**

EO 1.11 **Given a circuit showing current flows, IDENTIFY the polarity of the voltage drops in the circuit.**

Conventional and Electron Flow

The direction of electron flow is from a point of negative potential to a point of positive potential. The direction of positive charges, or holes, is in the opposite direction of electron flow. This flow of positive charges is known as *conventional flow*. All of the electrical effects of electron flow from negative to positive, or from a high potential to a lower potential, are the same as those that would be created by flow of positive charges in the opposite direction; therefore, it is important to realize that both conventions are in use, and they are essentially equivalent. In this manual, the electron flow convention is used.

Polarities

All voltages and currents have polarity as well as magnitude. In a series circuit, there is only one current, and its polarity is from the negative battery terminal through the rest of the circuit to the positive battery terminal. Voltage drops across loads also have polarities. The easiest way to find these polarities is to use the direction of the electron current as a basis. Then, where the electron current enters the load, the voltage is negative (Figure 31). This holds true regardless of the number or type of loads in the circuit. The drop across the load is opposite to that of the source. The voltage drops oppose the source voltage and reduce it for the other loads. This is because each load uses energy, leaving less energy for other loads.

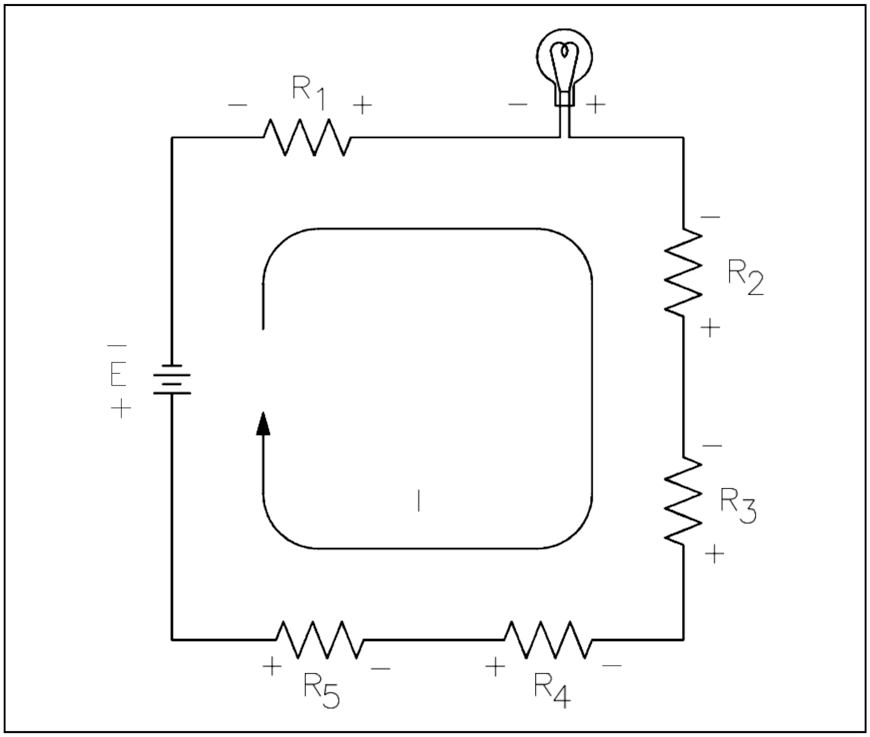

Figure 31 Voltage Polarities

Summary

The important information in this chapter is summarized below.

Voltage Polarity and Current Direction Summary

- The direction of electron flow is from a point of negative potential to a point of positive potential.
- The direction of positive charges, or holes, is in the opposite direction of electron flow. This flow of positive charges is known as "conventional flow."
- Where the electron current enters the load, the voltage is negative.

KIRCHHOFF'S LAWS

Kirchhoff's two laws reveal a unique relationship between current, voltage, and resistance in electrical circuits that is vital to performing and understanding electrical circuit analysis.

EO 1.12 STATE Kirchhoff's voltage law.

EO 1.13 STATE Kirchhoff's current law.

EO 1.14 Given a circuit, SOLVE problems for voltage and current using Kirchhoff's laws.

Kirchhoff's Laws

In all of the circuits examined so far, Ohm's Law described the relationship between current, voltage, and resistance. These circuits have been relatively simple in nature. Many circuits are extremely complex and cannot be solved with Ohm's Law. These circuits have many power sources and branches which would make the use of Ohm's Law impractical or impossible.

Through experimentation in 1857 the German physicist Gustav Kirchhoff developed methods to solve complex circuits. Kirchhoff developed two conclusions, known today as Kirchhoff's Laws.

Law 1: The sum of the voltage drops around a closed loop is equal to the sum of the voltage sources of that loop (Kirchhoff's Voltage Law).

Law 2: The current arriving at any junction point in a circuit is equal to the current leaving that junction (Kirchhoff's Current Law).

Kirchhoff's two laws may seem obvious based on what we already know about circuit theory. Even though they may seem very simple, they are powerful tools in solving complex and difficult circuits.

Kirchhoff's laws can be related to conservation of energy and charge if we look at a circuit with one load and source. Since all of the power provided from the source is consumed by the load, energy and charge are conserved. Since voltage and current can be related to energy and charge, then Kirchhoff's laws are only restating the laws governing energy and charge conservation.

The mathematics involved becomes more difficult as the circuits become more complex. Therefore, the discussion here will be limited to solving only relatively simple circuits.

Kirchhoff's Voltage Law

Kirchhoff's first law is also known as his "voltage law." The voltage law gives the relationship between the "voltage drops" around any closed loop in a circuit, and the voltage sources in that loop. The total of these two quantities is always equal. In equation form:

$$E_{source} = E_1 + E_2 + E_3 + \text{etc.} = I_1R_1 + I_2R_2 + I_3R_3 + \text{etc.}$$

$$\Sigma E_{source} = \Sigma IR \qquad (2\text{-}14)$$

where the symbol Σ (the Greek letter sigma) means "the sum of."

Kirchhoff's voltage law can be applied only to closed loops (Figure 32). A closed loop must meet two conditions:

1. It must have one or more voltage sources.
2. It must have a complete path for current flow from any point, around the loop, and back to that point.

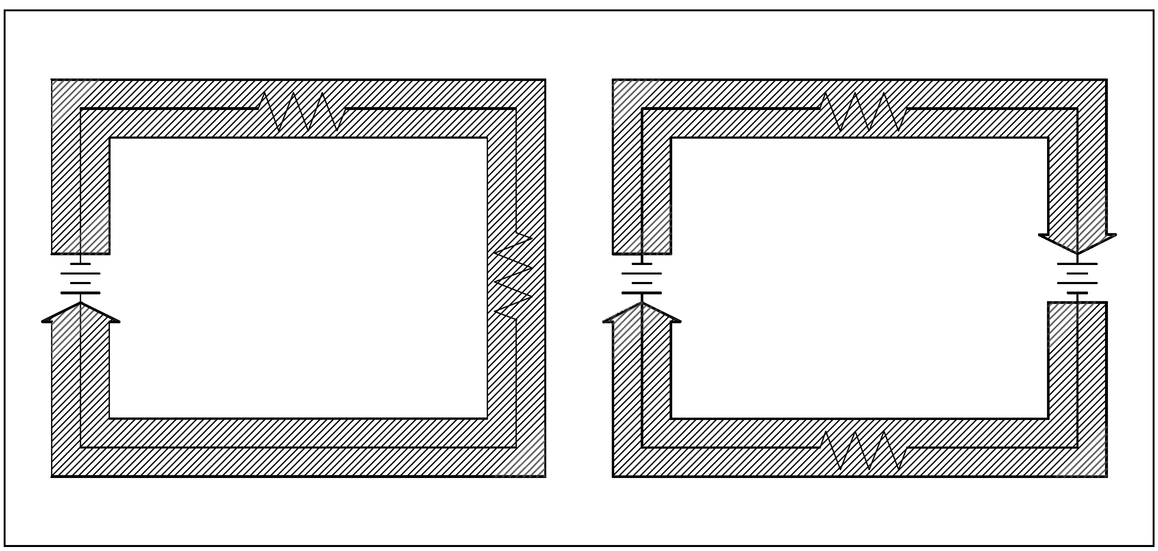

Figure 32 Closed Loop

You will remember that in a simple series circuit, the sum of the voltage drops around the circuit is equal to the applied voltage. Actually, this is Kirchhoff's voltage law applied to the simplest case, that is, where there is only one loop and one voltage source.

Applying Kirchhoff's Voltage Law

For a simple series circuit, Kirchhoff's voltage law corresponds to Ohm's Law. To find the current in a circuit (Figure 33) by using Kirchhoff's voltage law, use equation (2-15).

$$\Sigma E_{\text{source}} = \Sigma IR \qquad (2\text{-}15)$$

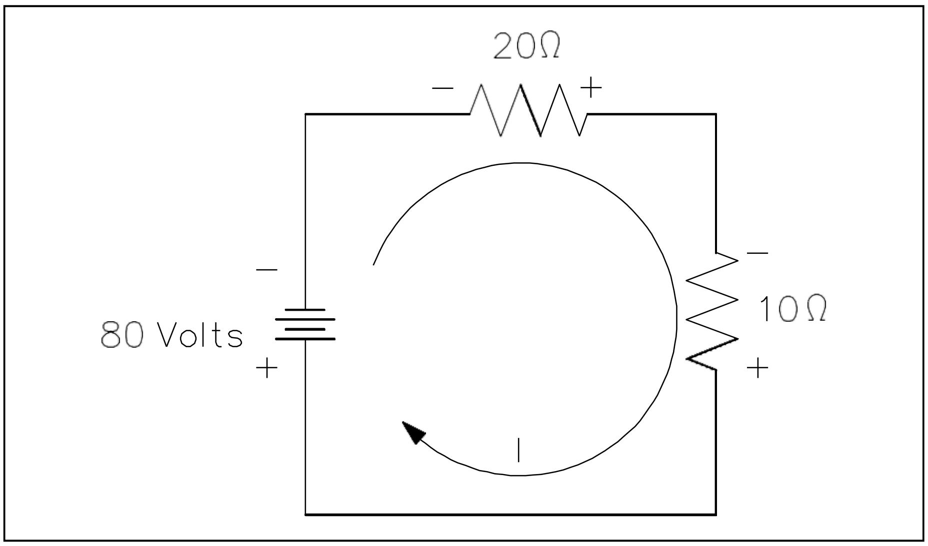

Figure 33 Using Kirchhoff's Voltage Law to find Current with one Source

$80 = 20(I) + 10(I)$

$80 = 30(I)$

$I = 80/30 = 2.66$ amperes

In the problem above, the direction of current flow was known before solving the problem. When there is more than one voltage source, the direction of current flow may or may not be known. In such a case, a direction of current flow must be assumed in the beginning of the problem. All the sources that would aid the current in the assumed direction of current flow are then positive, and all that would oppose current flow are negative. If the assumed direction is correct, the answer will be positive. The answer would be negative if the direction assumed was wrong. In any case, the correct magnitude will be attained.

For example, what is the current flow in Figure 34? Assume that the current is flowing in the direction shown.

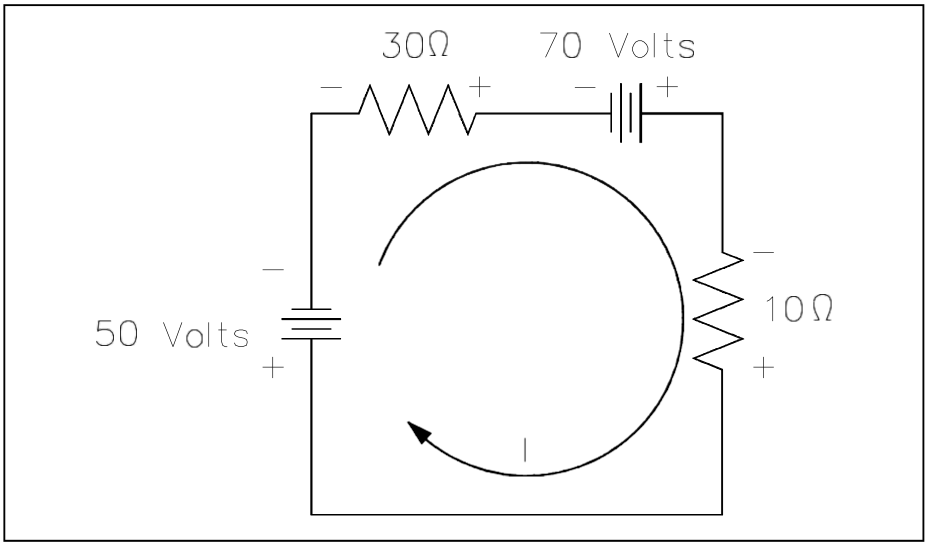

Figure 34 Using Kirchhoff's Voltage Law to find Current with Multiple Battery Sources

Using Kirchhoff's Voltage Law:

$$\sum E_{source} = \sum IR$$

$$50 - 70 = 30I + 10I$$

$$-20 = 40I$$

$$I = \frac{-20}{40}$$

$$I = -0.5$$

The result is negative. The current is actually 0.5 ampere in the opposite direction to that of the assumed direction.

Kirchhoff's Current Law

Kirchhoff's second law is called his current law and states: "At any junction point in a circuit, the current arriving is equal to the current leaving." Thus, if 15 amperes of current arrives at a junction that has two paths leading away from it, 15 amperes will divide among the two branches, but a total of 15 amperes must leave the junction. We are already familiar with Kirchhoff's current law from parallel circuits, that is, the sum of the branch currents is equal to the total current entering the branches, as well as the total current leaving the branches (Figure 35).

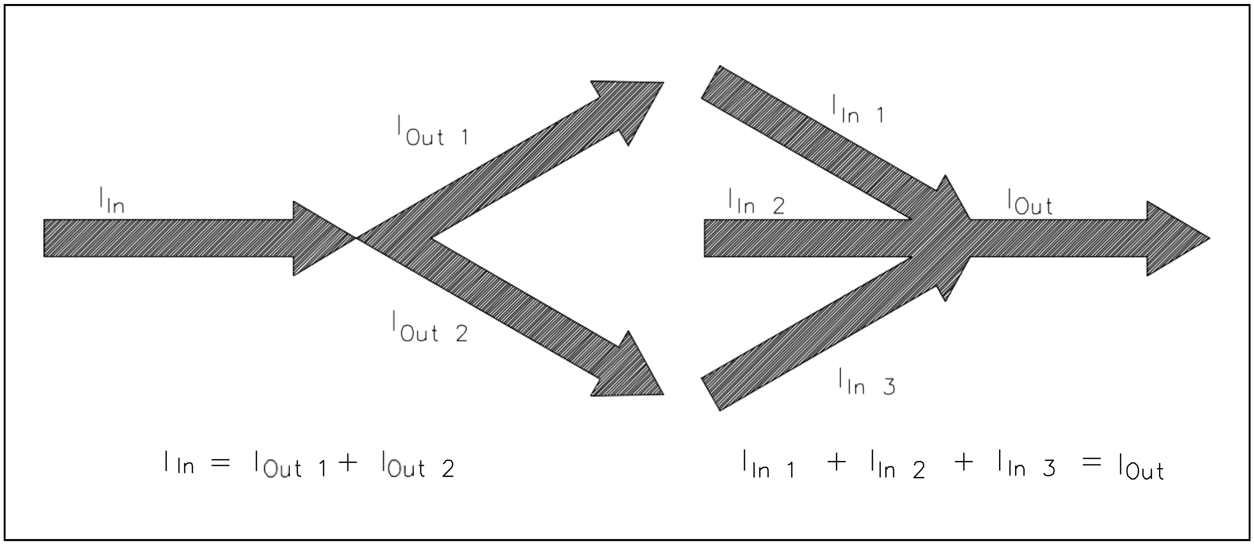

Figure 35 Illustration of Kirchhoff's Current Law

In equation form, Kirchhoff's current law may be expressed:

$$I_{IN} - I_{OUT} = 0 \qquad (2\text{-}16)$$

or

$$I_{IN} = I_{OUT}$$

Normally Kirchhoff's current law is not used by itself, but with the voltage law, in solving a problem.

Example: Find I_2 in the circuit shown in Figure 36 using Kirchhoff's voltage and current laws.

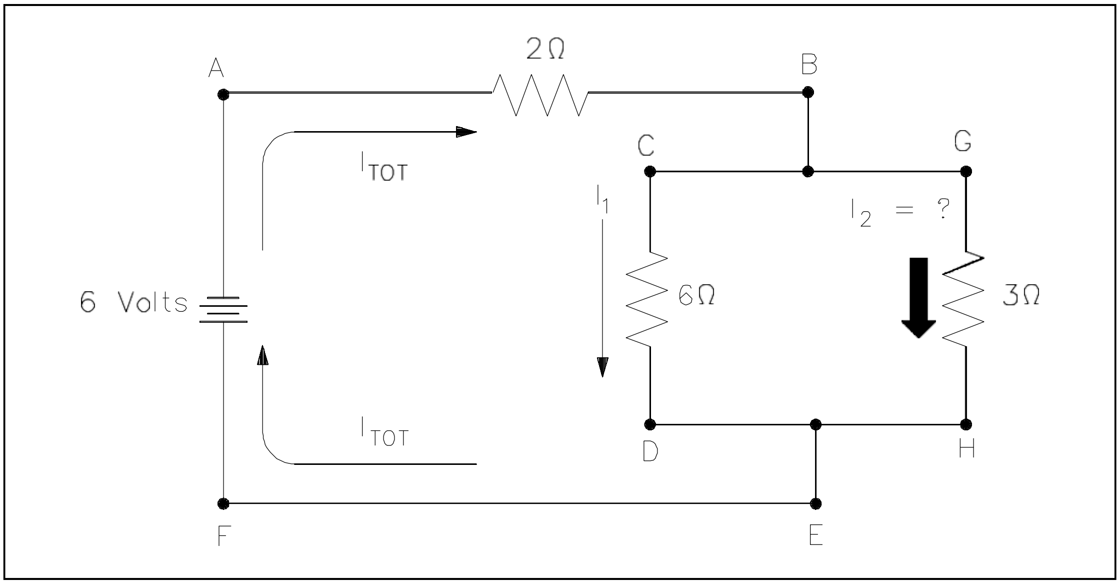

Figure 36 Using the Current Law

Solution:

First, apply Kirchhoff's voltage law to both loops.

Loop ABCDEF

$\sum IR = \sum E_{source}$
$2\ I_{total} + 6\ I_1 = 6$

Loop ABGHEF

$\sum IR = \sum E_{source}$
$2\ I_{total} + 3\ I_2 = 6$

Since Kirchhoff'a current law states $I_{total} = I_1 + I_2$, substitute $(I_1 + I_2)$ in the place of I_{total} in both loop equations and simplify.

Loop ABCDEF

$2\ (I_1 + I_2) + 6\ I_1 = 6$
$2\ I_1 + 2\ I_2 + 6\ I_1 = 6$
$8\ I_1 + 2\ I_2 = 6$

Loop ABGHEF

$2\ (I_1 + I_2) + 3\ I_2 = 6$
$2\ I_1 + 2\ I_2 + 3\ I_2 = 6$
$2\ I_1 + 5\ I_2 = 6$

We now have two equations and two unknowns and must eliminate I_1 to find I_2. One way is to multiply Loop ABGHEF equation by four, and subtract Loop ABCDEF equation from the result.

Multiply by 4:

$4\ (2\ I_1 + 5\ I_2 = 6)$
$8\ I_1 + 20\ I_2 + 24$

Subtract:

$$\begin{array}{rcrcr} 8I_1 & + & 20I_2 & = & 24 \\ -(8I_1 & + & 2I_2 & = & 6) \\ & & 18I_2 & = & 18 \end{array}$$

Now we have an equation with only I_2, which is the current we are looking for.

$$18I_2 = 18$$

$$I_2 = \frac{18}{18} = 1 \text{ ampere}$$

This circuit could have been solved simply by using Ohm's Law, but we used Kirchhoff's Laws to show the techniques used in solving complex circuits when Ohm's Law cannot be used.

Summary

The important information in this chapter is summarized below.

Kirchhoff's Laws Summary

- Kirchhoff's voltage law states that the sum of the voltage drops around a closed loop is equal to the sum of the voltage sources of that loop.
- Kirchhoff's current law states that the current arriving at any junction point in a circuit is equal to the current leaving that junction.
- Since voltage and current can be related to energy and charge, then Kirchhoff's laws are only restating the laws governing energy and charge conservation.

DC CIRCUIT ANALYSIS

All of the rules governing DC circuits that have been discussed so far can now be applied to analyze complex DC circuits. To apply these rules effectively, loop equations, node equations, and equivalent resistances must be used.

EO 1.15 **Given a simple DC circuit, DETERMINE the equivalent resistance of series and parallel combinations of elements.**

Loop Equations

As we have already learned, Kirchhoff's Laws provide a practical means to solve for unknowns in a circuit. Kirchhoff's current law states that at any junction point in a circuit, the current arriving is equal to the current leaving. In a series circuit the current is the same at all points in that circuit. In parallel circuits, the total current is equal to the sum of the currents in each branch. Kirchhoff's voltage law states that the sum of all potential differences in a closed loop equals zero.

Using Kirchhoff's laws, it is possible to take a circuit with two loops and several power sources (Figure 37) and determine loop equations, solve loop currents, and solve individual element currents.

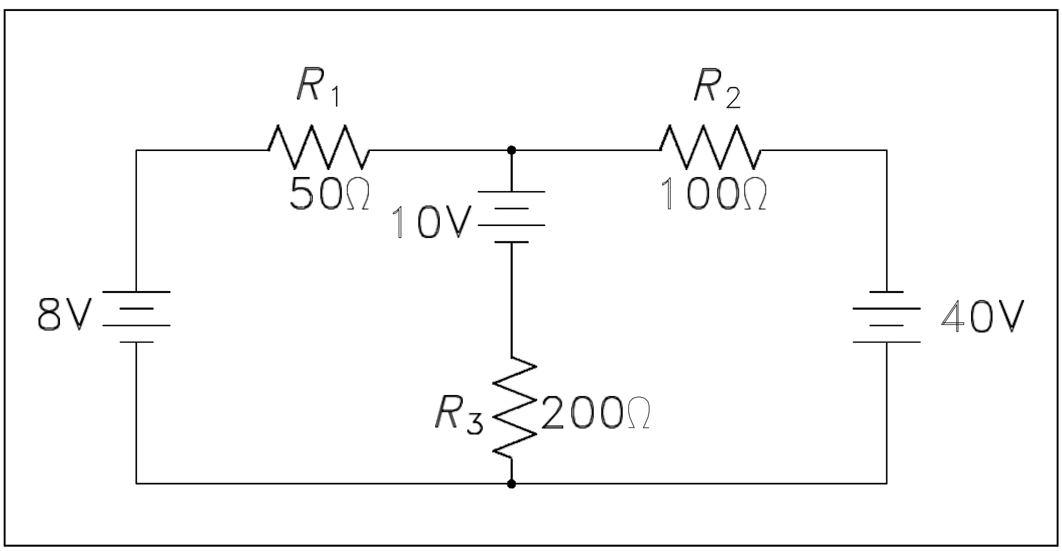

Figure 37 Example Circuit for Loop Equations

The first step is to draw an assumed direction of current flow (Figure 38). It does not matter whether the direction is correct. If it is wrong, the resulting value for current will be negative.

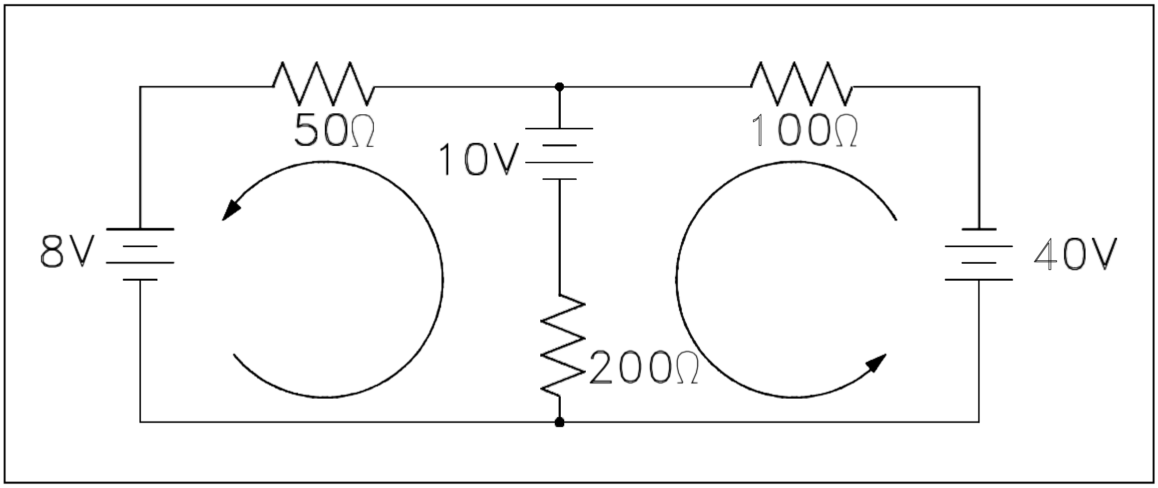

Figure 38 Assumed Direction of Current Flow

Second, mark the polarity of voltage across each component (Figure 39). It is necessary to choose a direction for current through the center leg, but it is not necessary to put in a new variable. It is simply $I_2 - I_1$.

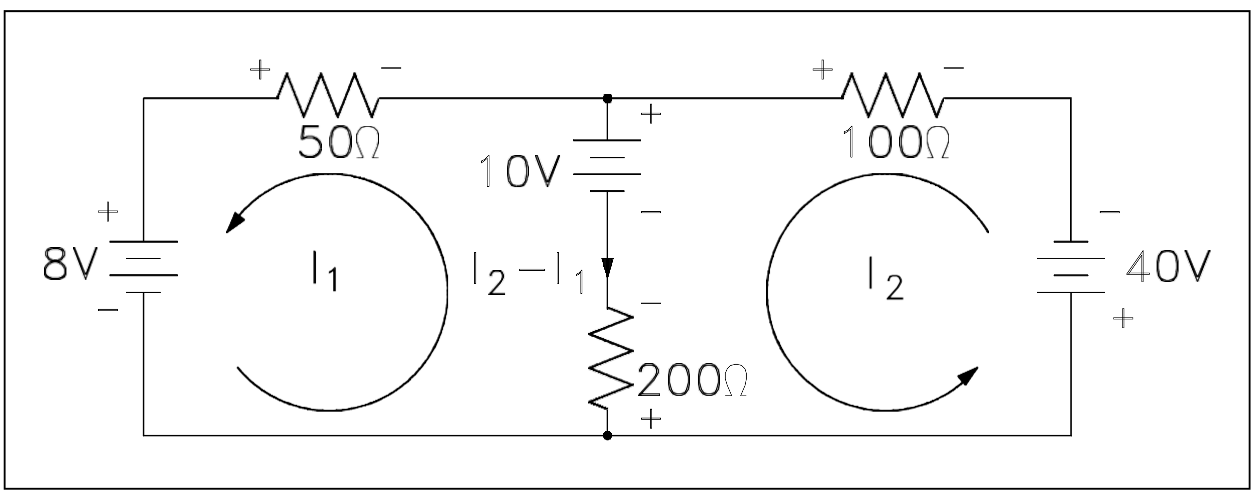

Figure 39 Marking Polarity

Third, apply Kirchhoff's voltage law to loops one and two by picking a point in each loop and writing a loop equation of the voltage drops around the loop; then set the equation equal to zero.

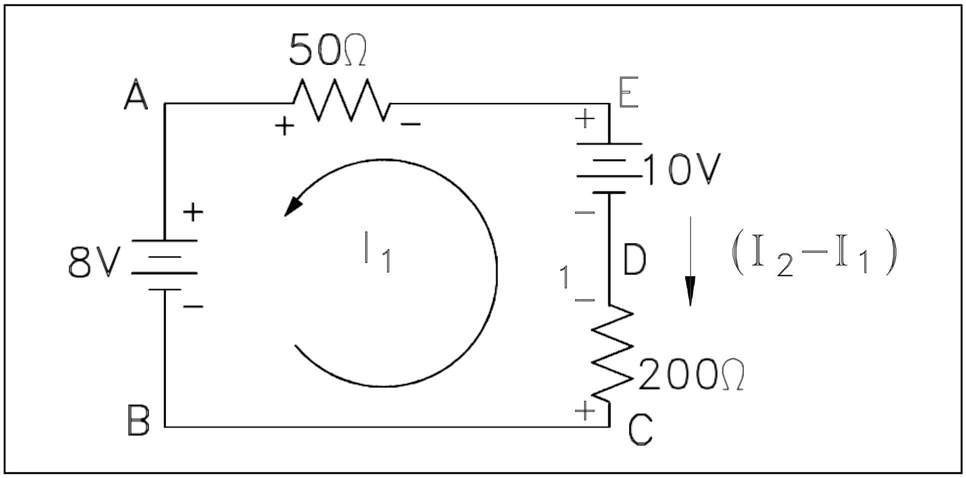

Figure 40 Applying Voltage Law to Loop 1

Figure 40 shows Loop one.

From Point A to Point B, there is an increase in voltage of 8 volts. From Point C to Point D, there is an increase in voltage of 200 (I_2 - I_1). From Point D to Point E, there is a decrease in voltage of 10 volts. From Point E to Point A, there is a voltage decrease of $50I_1$ volts. The result in equation form is illustrated in equation (2-16).

$$8 + 200 (I_2 - I_1) - 50 I_1 - 10 = 0 \tag{2-17}$$

Using the same procedure for Loop 2 of Figure 39, the resulting equation is shown in equation (2-18).

$$10 - 200 (I_2 - I_1) + 40 - 100 I_2 = 0 \tag{2-18}$$

Fourth, solve equations (2-17) and (2-18) simultaneously. First, rearrange and combine like terms in the equation for Loop 1.

$$-50 I_1 + 200 I_2 - 200 I_1 = 10 - 8$$
$$-250 I_1 + 200 I_2 = 2$$

Divide both sides by two.

$$-125 I_1 + 100 I_2 = 1$$

Rearrange and combine like terms in the Loop 2 equation.

$$-200\ I_2 + 200\ I_1 - 100\ I_2 = -10 - 40$$
$$200\ I_1 - 300\ I_2 = -50$$

Multiplying the Loop 1 equation by 3, and add it to the Loop 2 equation.

$$3(-125\ I_1 + 100\ I_2 = 1) = -375\ I_1 + 300I_2 = 3$$
$$+\ 200\ I_2 - 300I_2 = -50$$
$$-175\ I_1 = -47$$

Solving for I_1:

$$-175\ I_1 = -47$$
$$I_1 = \frac{-47}{-175} = 0.2686\ \text{amp} = 268.6\ \text{mA}$$

Solving for I_2 using the Loop 1 equation:

$$-125\ (0.2686) + 100\ I_2 = 1$$
$$100\ I_2 = 1 + 33.58$$
$$I_2 = \frac{34.58}{100}$$
$$I_2 = 0.3458\ \text{amp} = 345.8\ \text{mA}$$

The current flow through R_1 (50Ω) is I_1. The current flow through R_2(100Ω) is I_2, and through R_3(200Ω) is I_2 - I_1:

$$I_3 = I_2 - I_1 = 345.8\ \text{mA} - 268.6\ \text{mA}$$
$$I_3 = I_2 - I_1 = 77.2\ \text{mA}$$

Fifth, apply Ohm's Law to obtain the voltage drops across Resistors R_1, R_2, and R_3:

$V_1 = I_1R_1 = (0.2686 \text{ amps})(50\Omega) = 13.43 \text{ Volts}$

$V_2 = I_2R_2 = (0.3458 \text{ amps})(100\Omega) = 34.58 \text{ Volts}$

$V_3 = (I_2 - I_1)\, R_3 = (0.0772 \text{ amps})(200\Omega) = 15.44 \text{ Volts}$

Sixth, check the calculations by applying Kirchhoff's Laws:

Check 1: Apply Kirchhoff's voltage law to the larger outer loop (Figure 41).

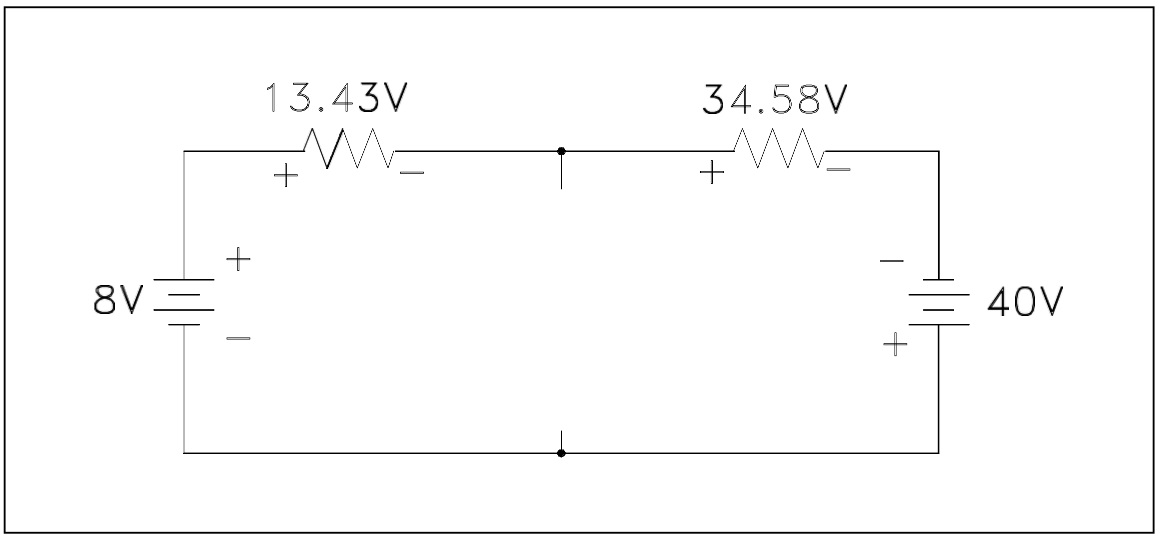

Figure 41 Applying Voltage Laws to Outer Loop

The sum of the voltage drops around the loop is essentially zero. (Not exactly zero due to rounding off.)

$$\begin{aligned} 8 - 13.43 - 34.58 + 40 &= 0 \\ -0.01 &\cong 0 \end{aligned}$$

Therefore, the solution checks.

Check 2: Use Kirchhoff's current law at one of the junctions (Figure 42).

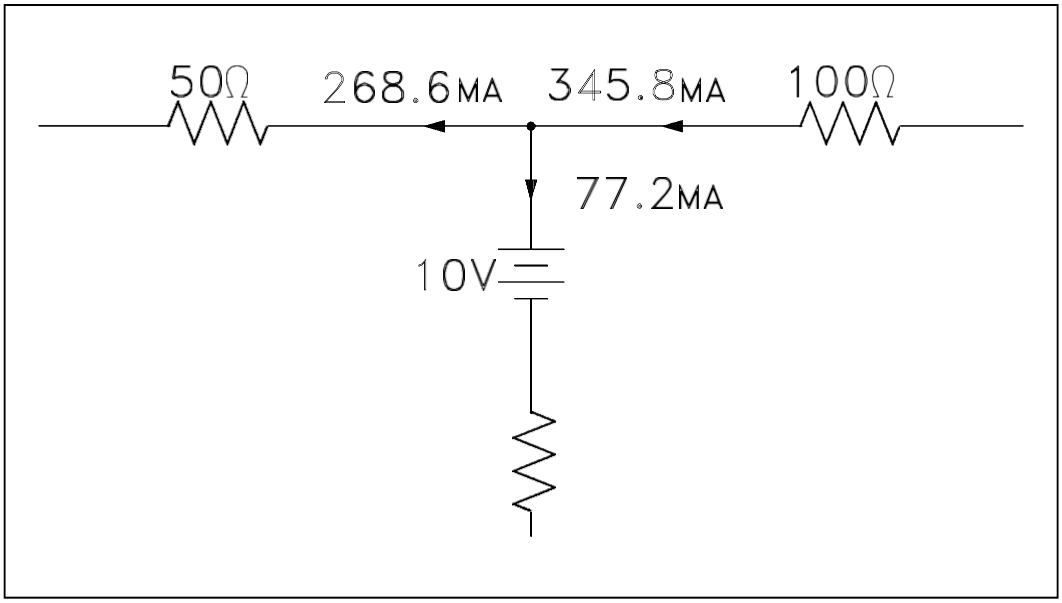

Figure 42 Applying Current Law to Junction

The sum of the currents out of the junction is:

$$0.2686 + 0.0772 = 0.3458 \text{ a}$$
$$= 345.8 \text{ ma}$$

The current into the junction is 345.8 ma.

The current into the junction is equal to the current out of the junction. Therefore, the solution checks.

Node Equations

Kirchhoff's current law, as previously stated, says that at any junction point in a circuit the current arriving is equal to the current leaving. Let us consider five currents entering and leaving a junction shown as P (Figure 43). This junction is also considered a node.

Assume that all currents entering the node are positive, and all currents that leave the node are negative. Therefore, I_1, I_3, and I_4 are positive, and I_2 and I_5 are negative. Kirchhoff's Law also states that the sum of all the currents meeting at the node is zero. For Figure 43, Equation (2-19) represents this law mathematically.

$$I_1 + I_2 + I_3 + I_4 + I_5 = 0 \qquad (2\text{-}19)$$

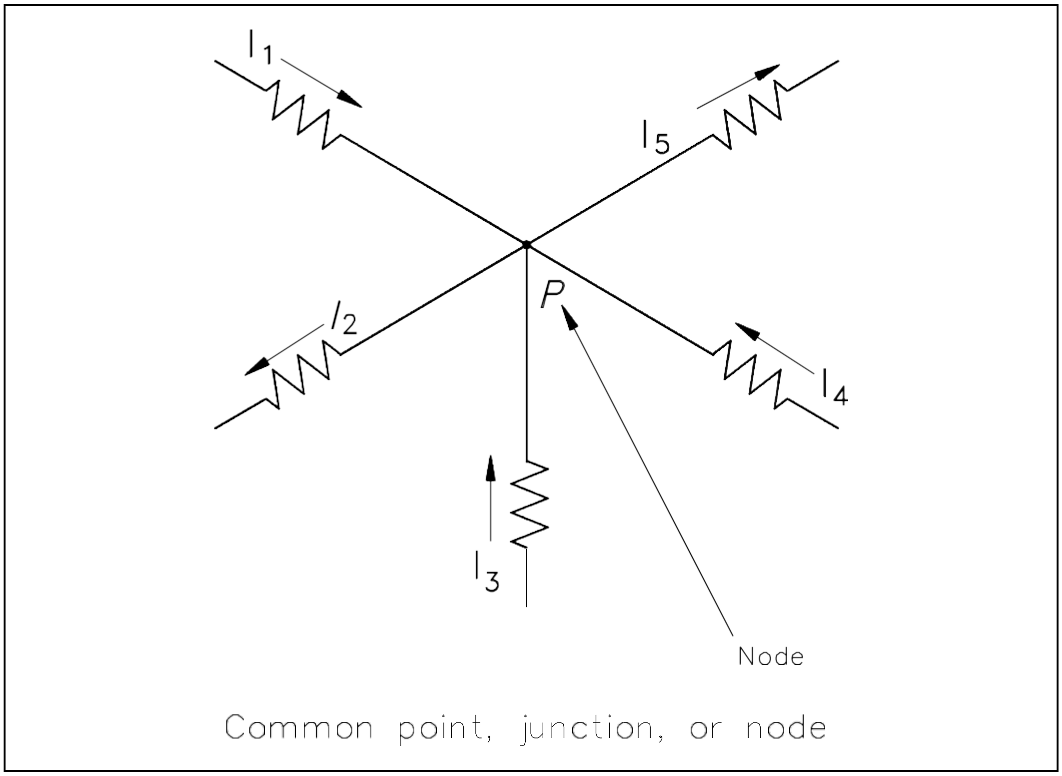

Figure 43 Node Point

By solving node equations, we can calculate the unknown node voltages. To each node in a circuit we will assign a letter or number. In Figure 44, A, B, C, and N are nodes, and N and C are principal nodes. Principal nodes are those nodes with three or more connections. Node C will be our selected reference node. V_{AC} is the voltage between Nodes A and C; V_{BC} is the voltage between Nodes B and C; and V_{NC} is the voltage between Nodes N and C. We have already determined that all node voltages have a reference node; therefore, we can substitute V_A for V_{AC}, V_B for V_{BC}, and V_N for V_{NC}.

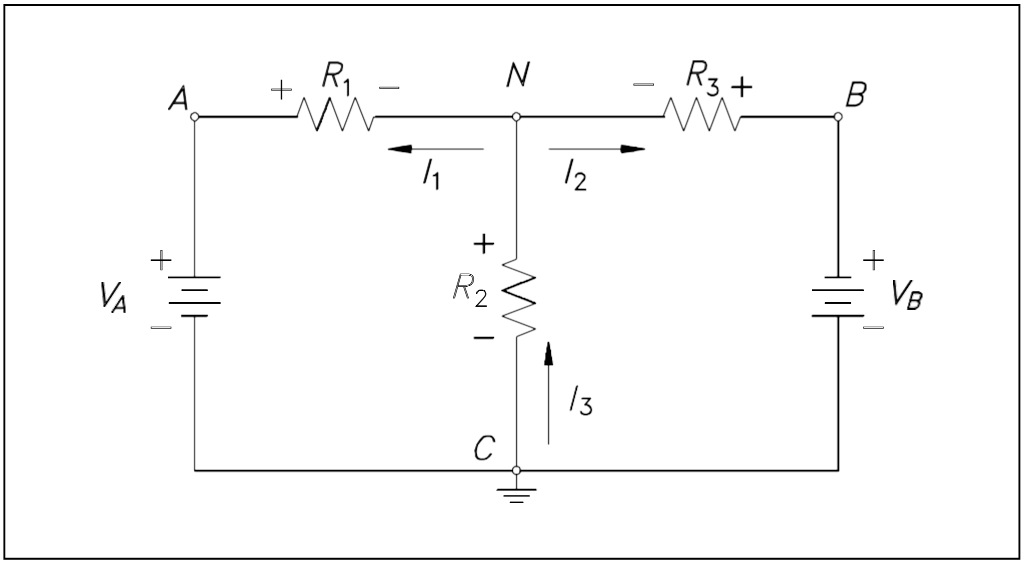

Figure 44 Circuit for Node Analysis

Assume that loop currents I_1 and I_2 leave Node N, and that I_3 enters Node N (Figure 44).

From Kirchhoff's current law:

$$\begin{aligned} \sum I &= 0 \\ I_1 + I_2 + I_3 &= 0 \qquad (2\text{-}20) \\ I_3 &= I_1 + I_2 \end{aligned}$$

Using Ohm's Law and solving for the current through each resistor we obtain the following.

$I = \dfrac{V_R}{R}$ where V_R is the voltage across resistor, R.

$$I_3 = \frac{V_N}{R_2}$$

$$I_1 = \frac{V_A - V_N}{R_1}$$

$$I_2 = \frac{V_B - V_N}{R_3}$$

Substitute these equations for I_1, I_2, and I_3 into Kirchhoff's current equation (2-20) yields the following.

$$\frac{V_N}{R_2} = \frac{V_A - V_N}{R_1} + \frac{V_B - V_N}{R_3}$$

The circuit shown in Figure 45 can be solved for voltages and currents by using the node-voltage analysis.

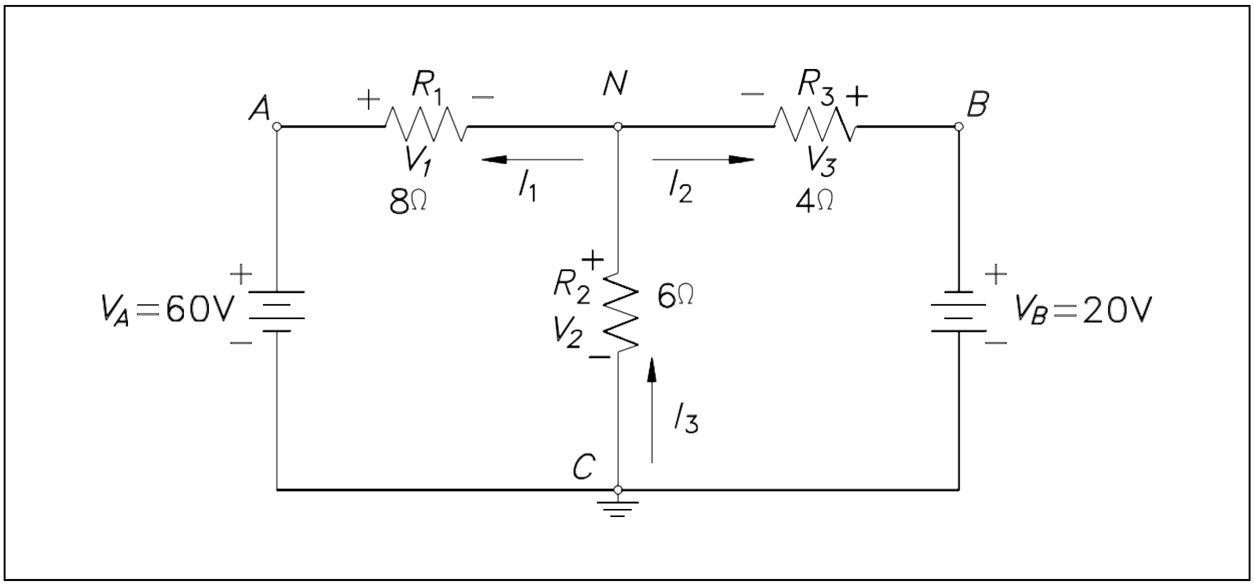

Figure 45 Node - Voltage Analysis

First, assume direction of current flow shown. Mark nodes A, B, C, and N, and mark the polarity across each resistor.

Second, using Kirchhoff's current law at Node N, solve for V_N.

$$I_3 = I_1 + I_2$$

$$\frac{V_N}{R_2} = \frac{V_A - V_N}{R_1} + \frac{V_B - V_N}{R_3}$$

$$\frac{V_N}{6} = \frac{60 - V_N}{8} + \frac{20 - V_N}{4}$$

Clear the fraction so that we have a common denominator:

$$4V_N = 3\ (60 - V_N) + 6\ (20 - V_N)$$
$$4V_N = 180 - 3V_N + 120 - 6V_N$$
$$13V_N = 300$$
$$V_N = 23.077$$

Third, find all voltage drops and currents.

$$V_1 = V_A - V_N = 60 - 23.077 = 36.923 \text{ Volts}$$
$$V_2 = V_N = 23.077 \text{ Volts}$$
$$V_3 = V_B - V_N = 20 - 23.077 = -3.077 \text{ Volts}$$

The negative value for V_3 shows that the current flow through R_3 is opposite that which was assumed and that the polarity across R_3 is reversed.

$$I_1 = \frac{V_1}{R_1} = \frac{36.923\ \text{V}}{8\Omega} = 4.65 \text{ amps}$$
$$I_2 = \frac{V_3}{R_3} = \frac{-3.077\ \text{V}}{4\Omega} = -0.769 \text{ amps}$$
$$I_3 = \frac{V_2}{R_2} = \frac{23.077\ \text{V}}{6\Omega} = 3.846 \text{ amps}$$

The negative value for I_3 shows that the current flow through R_3 is opposite that which was assumed.

Series-Parallel Circuit Analysis

When solving for voltage, current, and resistance in a series-parallel circuit, follow the rules which apply to the series part of the circuit, and follow the rules which apply to the parallel part of the circuit. Solving these circuits can be simplified by reducing the circuit to a single equivalent resistance circuit, and redrawing the circuit in simplified form. The circuit is then called an equivalent circuit (Figure 46).

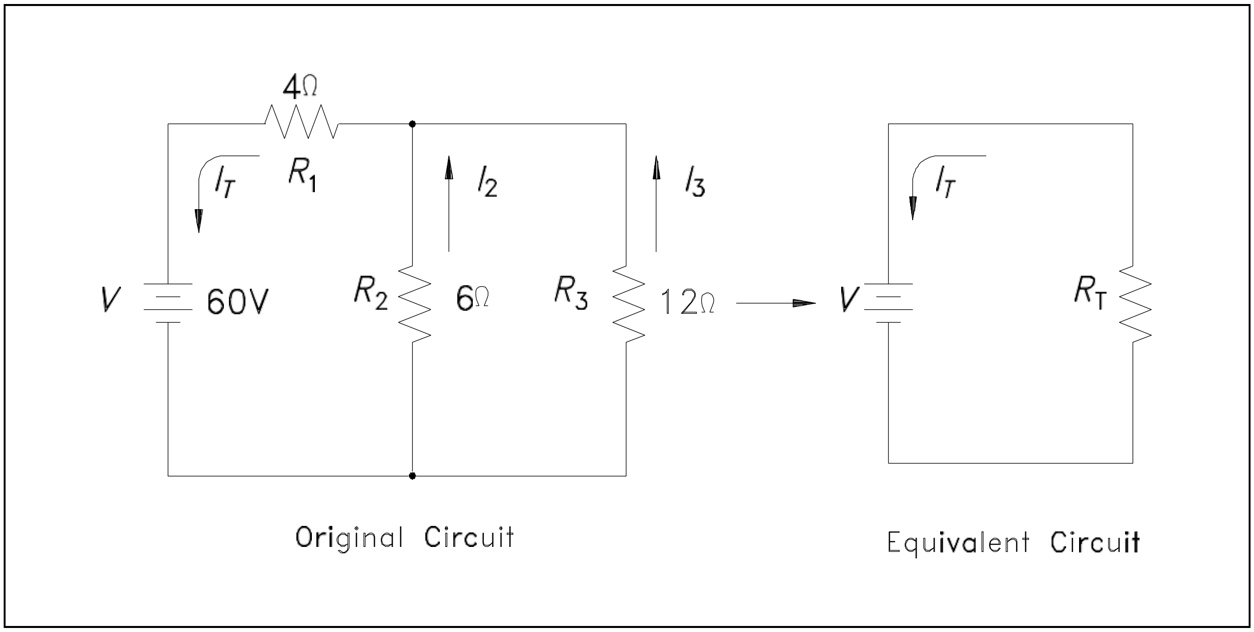

Figure 46 Redrawn Circuit Example

The easiest way to solve these types of circuits is to do it in steps.

Step 1: Find the equivalent resistance of the parallel branch:

$$R_p = \frac{R_2 R_3}{R_2 + R_3} = \frac{(6)(12)}{6 + 12} = \frac{72}{18} = 4\Omega$$

Step 2: Find the resistance of the equivalent series circuit:

$$R_T = R_1 + R_P = 4\Omega + 4\Omega = 8\Omega$$

Step 3: Find total current (I_T):

$$I_T = \frac{V}{R_T} = \frac{60\ V}{8\Omega} = 7.5 \text{ amps}$$

Step 4: Find I_2 and I_3. The voltage across R_1 and R_2 is equal to the applied voltage (V), minus the voltage drop across R_1.

$$V_2 = V_3 = V - I_T R_1 = 60 - (7.5 \text{ X } 4) = 30 \text{ V}$$

Then, I_2 and I_3 are calculated.

$$I_2 = \frac{V_2}{R_2} = \frac{30}{6} = 5 \text{ amps}$$

$$I_3 = \frac{V_3}{R_3} = \frac{30}{12} = 2.5 \text{ amps}$$

Y and Delta Network Calculation

Because of its shape, the network shown in Figure 47 is called a T (tee) or Y (wye) network. These are different names for the same network.

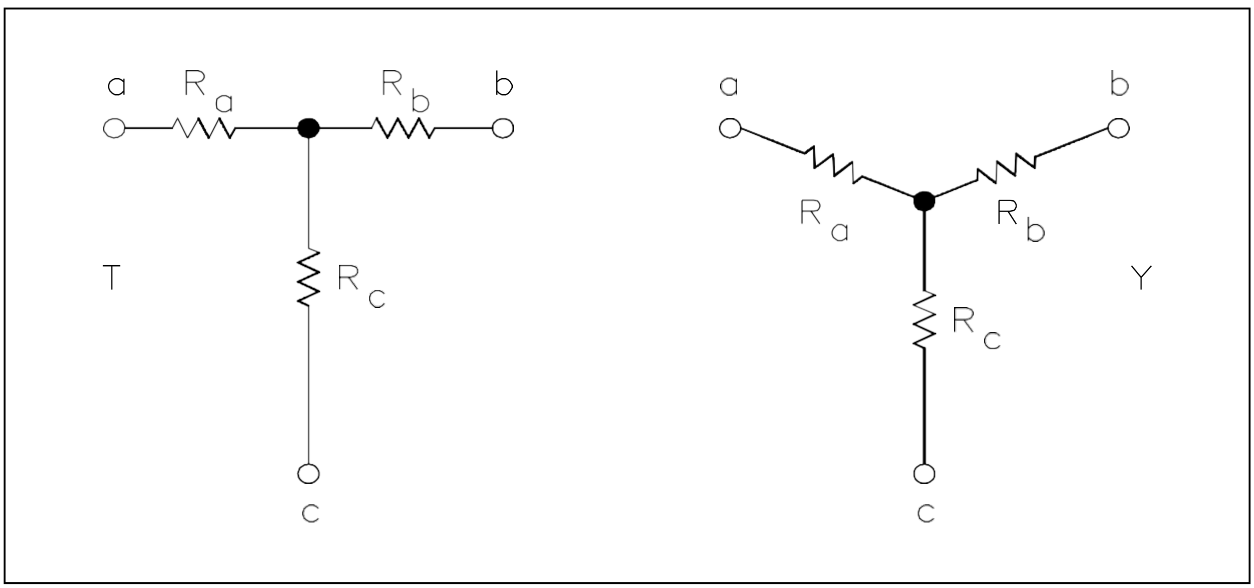

Figure 47 T or Y Network

The network shown in Figure 48 is called π (pi) or Δ (delta) because the shapes resemble Greek letters π and Δ. These are different names for the same network.

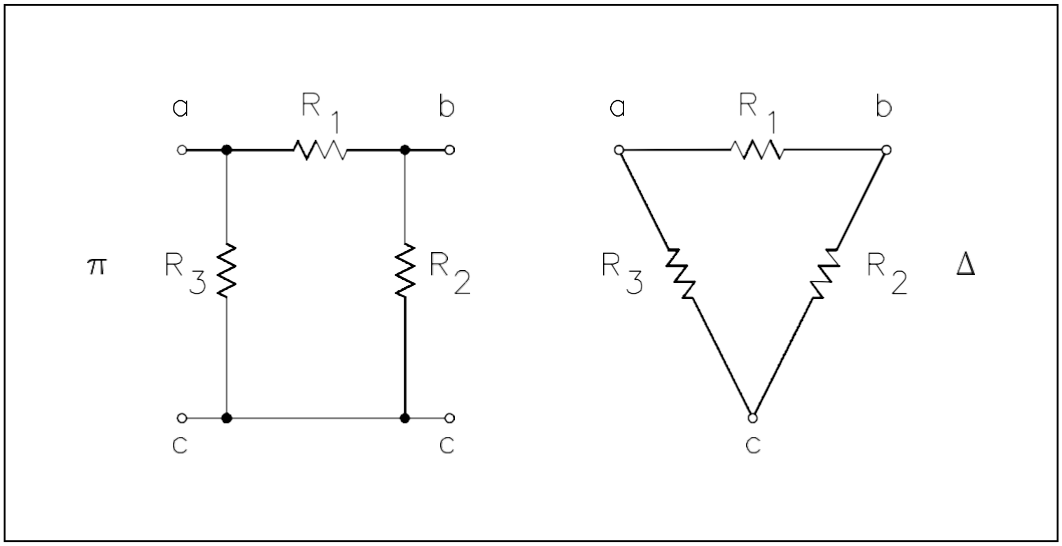

Figure 48 π (pi) or Δ (delta) Network

In order to analyze the circuits, it may be helpful to convert Y to Δ, or Δ to Y, to simplify the solution. The formulas that will be used for these conversions are derived from Kirchhoff's laws. The resistances in these networks are shown in a three-terminal network. After we use the conversion formulas, one network is equivalent to the other because they have equivalent resistances across any one pair of terminals (Figure 49).

Δ to Y conversion:

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

Rule 1: The resistance of any branch of a Y network is equal to the product of the two adjacent sides of a Δ network, divided by the sum of the three Δ resistances.

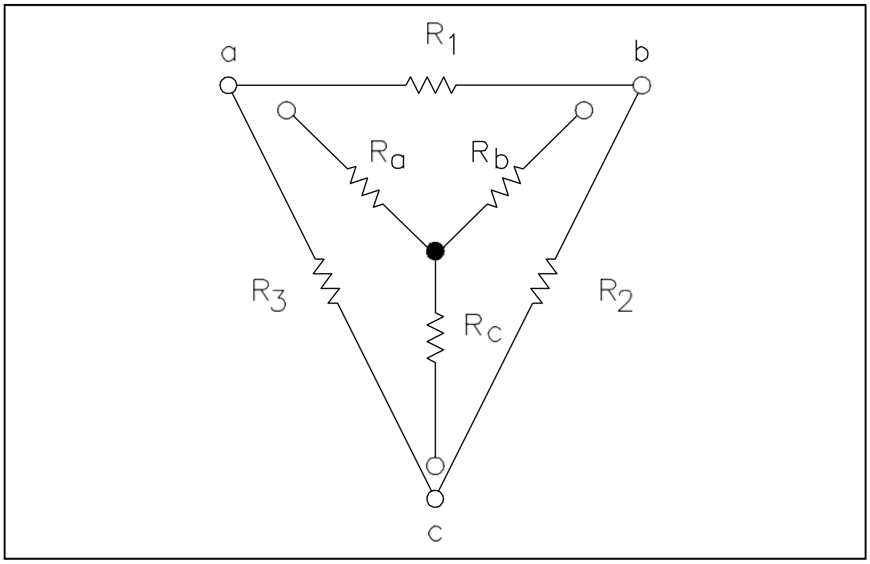

Figure 49 Y - Δ Equivalent

Y to Δ conversion:

$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

Rule 2: The resistance of any side of a Δ network is equal to the sum of the Y network resistance, multiplied in pairs, divided by the opposite branch of the Y network.

Let us consider a bridge circuit (Figure 50).

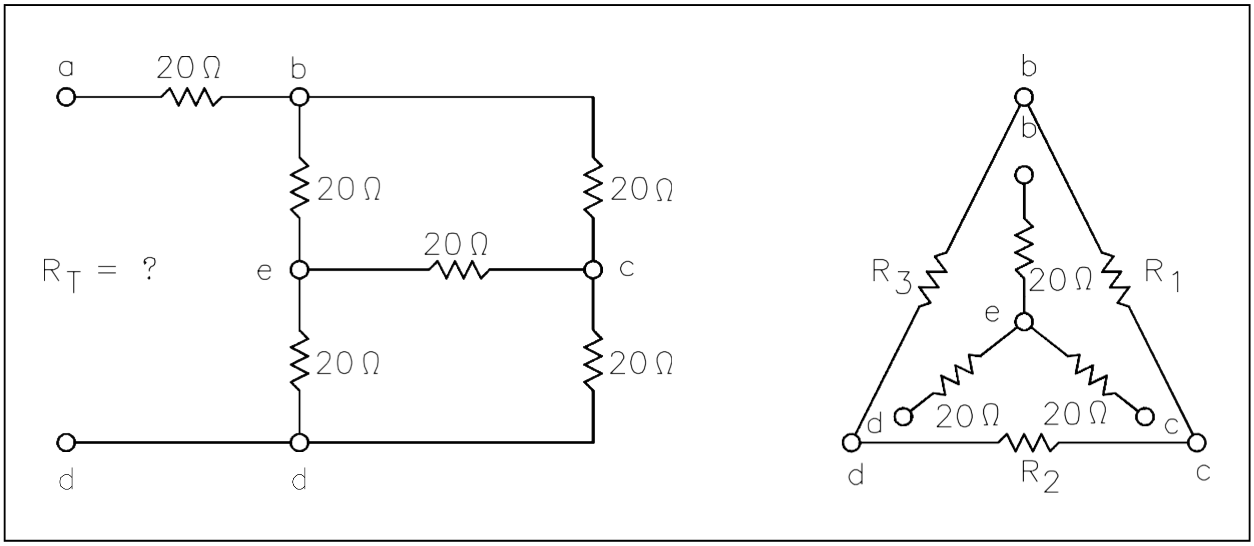

Figure 50 Bridge Circuit

Find R_t at terminals a and d.

Step 1: Convert the Y network (b-e, e-c, e-d) to the equivalent Δ network.

Using Rule 2:

$$R_1 = \frac{(20)(20) + (20)(20) + (20)(20)}{20} = \frac{1200}{20} = 60\Omega$$

$$R_2 = \frac{1200}{20} = 60\Omega$$

$$R_3 = \frac{1200}{20} = 60\Omega$$

Step 2: Now, we can redraw the Y circuit as a Δ circuit and reconnect it to the original circuit (Figure 51):

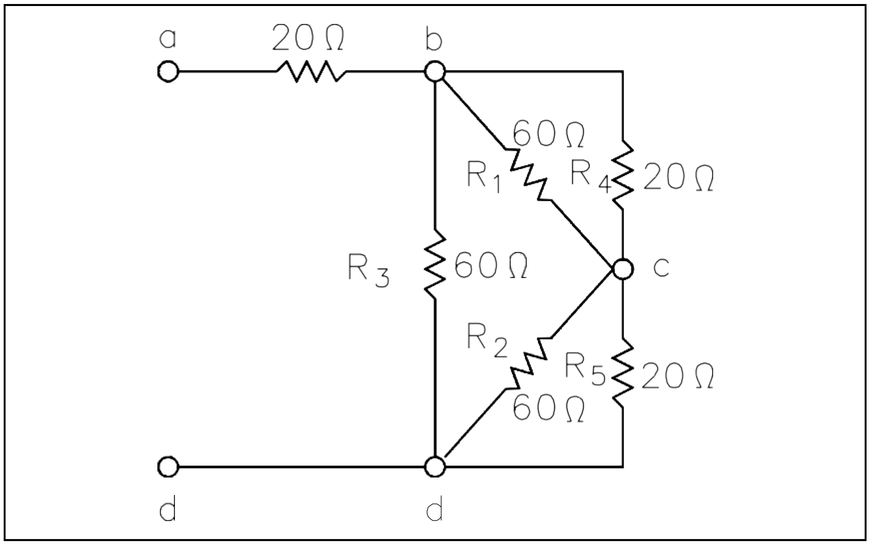

Figure 51 Y - Δ Redrawn Circuit

Step 3: Reduce and simplify the circuit. Note that the 20Ω and 60Ω branches are in parallel in Figure 51. Refer to Figures 51 and 52 for redrawing the circuit in each step below.

$$R_P = \frac{R_1 R_4}{R_1 + R_4} = \frac{(20)(60)}{20 + 60} = \frac{1200}{80} = 15\Omega$$

$$R_q = \frac{R_1 R_5}{R_1 + R_5} = \frac{(20)(60)}{20 + 60} = \frac{1200}{80} = 15\Omega$$

$$R_r = \frac{R_3(R_p + R_Q)}{R_3 + (R_p + R_Q)} = \frac{(60)(15 + 15)}{60 + 30} = \frac{1800}{90} = 20\Omega$$

$$R_T = 20 + 20 = 40\Omega$$

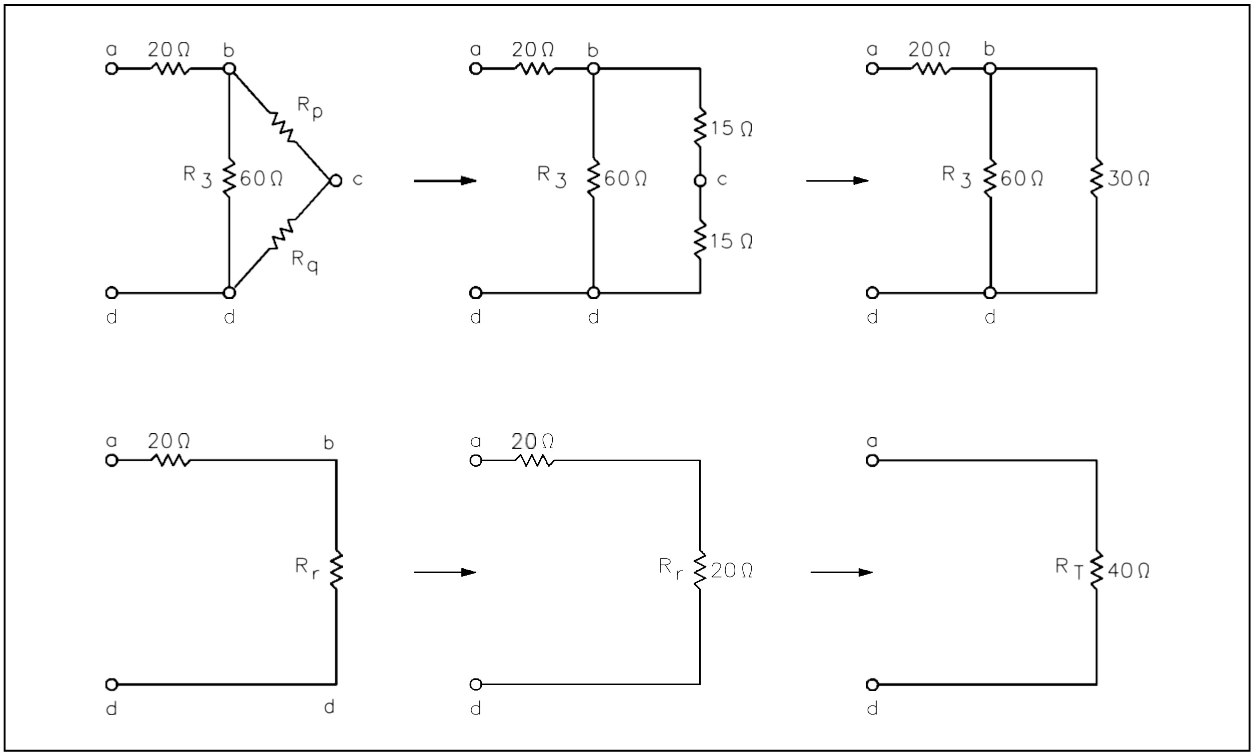

Figure 52 Steps to Simplify Redrawn Circuit

Summary

The important information in this chapter is summarized below.

DC Circuit Analysis Summary

- The current flow at any element in a DC circuit can be determined using loop equations.
- The voltage at any point in a DC circuit can be determined using node equations.
- The equivalent resistance of series and parallel combinations of elements can be used to simplify DC circuit analysis.

DC CIRCUIT FAULTS

Faults within a DC circuit will cause various effects, depending upon the nature of the fault. An understanding of the effects of these faults is necessary to fully understand DC circuit operation.

EO 1.16 DESCRIBE the voltage and current effects of an open in a DC circuit.

EO 1.17 DESCRIBE the voltage and current effects in a shorted DC circuit.

Open Circuit (Series)

A circuit must have a "complete" path for current flow, that is, from the negative side to the positive side of a power source. A series circuit has only one path for current to flow. If this path is broken, no current flows, and the circuit becomes an open circuit (Figure 53).

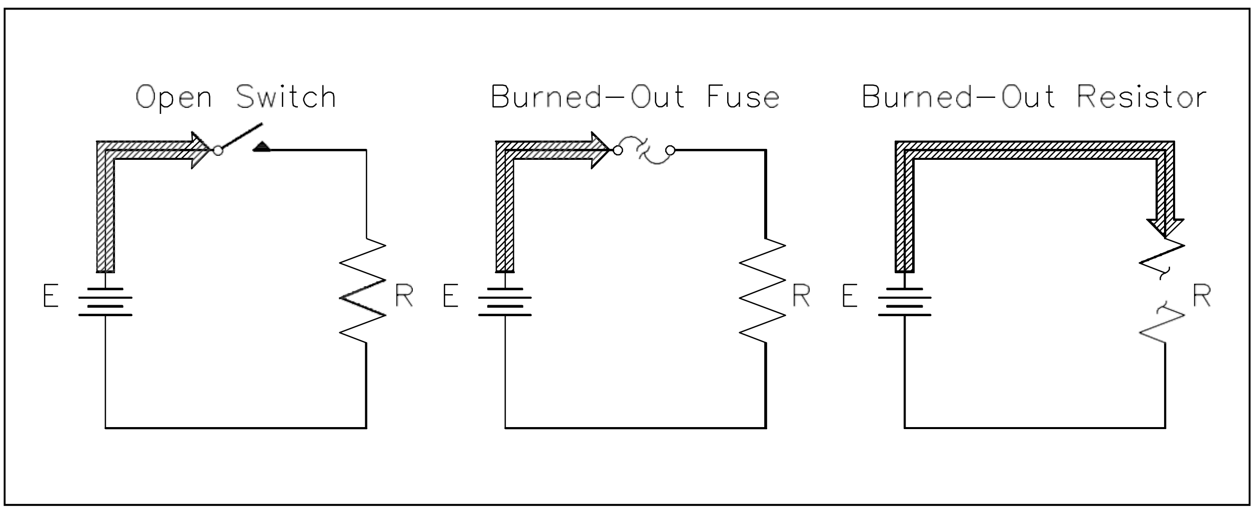

Figure 53 Open Series Circuit

Circuits can be opened deliberately, such as by the use of a switch, or they may be opened by a defect, such as a broken wire or a burned-out resistor.

Since no current flows in an open series circuit, there are no voltage drops across the loads. No power is consumed by the loads, and total power consumed by the circuit is zero.

Open Circuit (Parallel)

A parallel circuit has more than one path for current to flow. If one of the paths is opened, current will continue to flow as long as a complete path is provided by one or more of the remaining paths. It does not mean that you cannot stop current flow through a parallel circuit by opening it at one point; it means that the behavior of a parallel circuit depends on where the opening occurs (Figure 54).

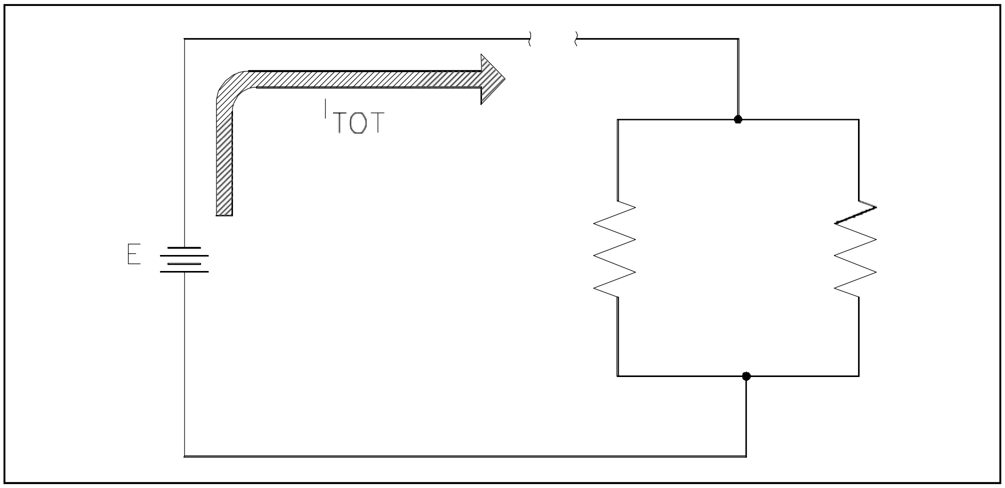

Figure 54 Open Parallel Circuit - Total

If a parallel circuit is opened at a point where only a branch current flows, then only that branch is open, and current continues to flow in the rest of the circuit (Figure 55).

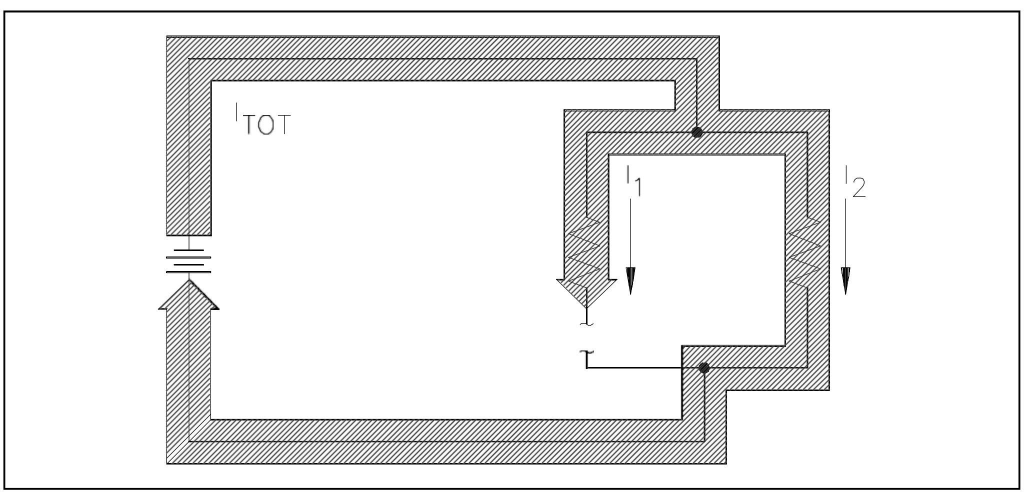

Figure 55 Open Parallel Circuit - Branch

Short Circuit (Series)

In a DC circuit, the only current limit is the circuit resistance. If there is no resistance in a circuit, or if the resistance suddenly becomes zero, a very large current will flow. This condition of very low resistance and high current flow is known as a "short circuit" (Figure 56).

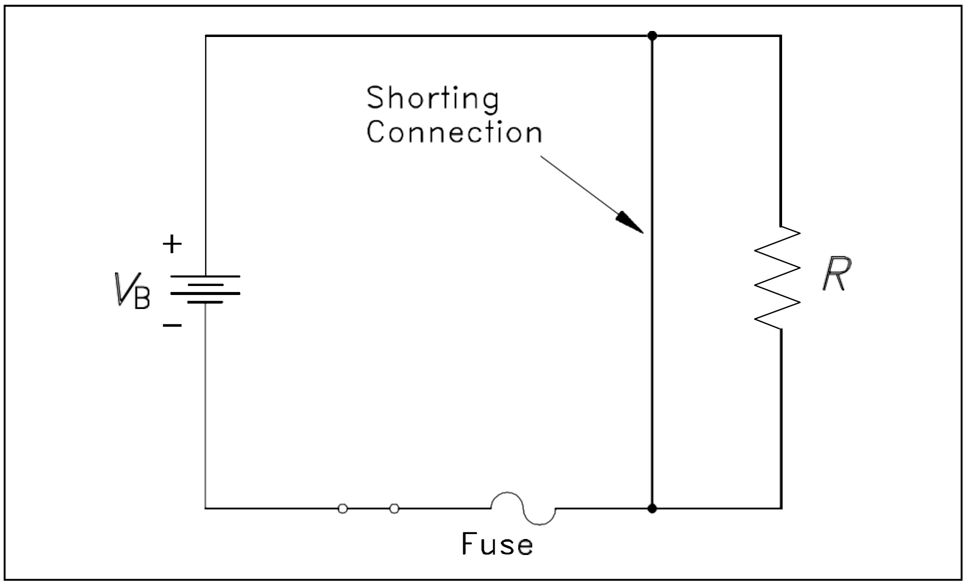

Figure 56 Shorted DC Circuit

A short circuit is said to exist if the circuit resistance is so low that current increases to a point where damage can occur to circuit components. With an increase in circuit current flow, the terminal voltage of the energy source will decrease. This occurs due to the internal resistance of the energy source causing an increased voltage drop within the energy source. The increased current flow resulting from a short circuit can damage power sources, burn insulation, and start fires. Fuses are provided in circuits to protect against short circuits.

Short Circuit (Parallel)

When a parallel circuit becomes short circuited, the same effect occurs as in a series circuit: there is a sudden and very large increase in circuit current (Figure 57).

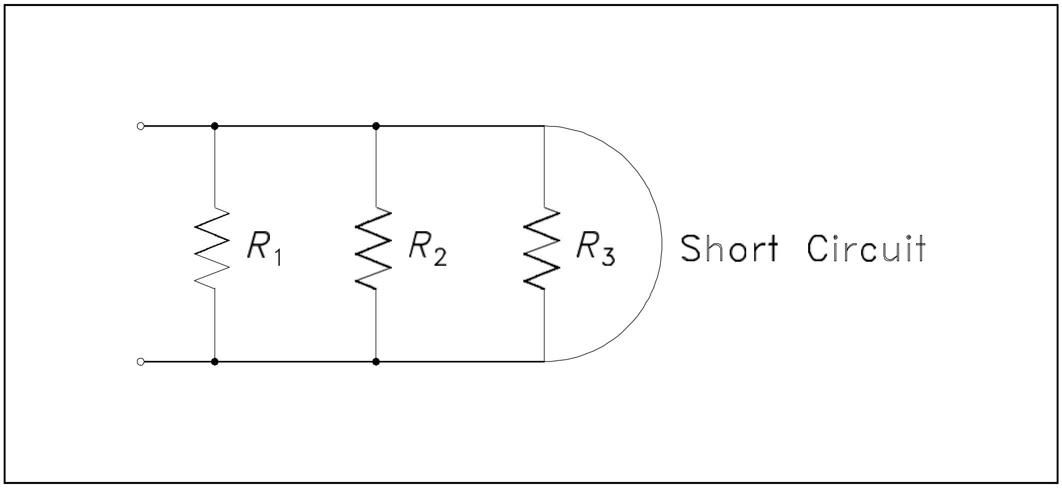

Figure 57 Shorted Parallel Circuit

Parallel circuits are more likely than series circuits to develop damaging short circuits. This is because each load is connected directly across the power source. If any of the load becomes shorted, the resistance between the power source terminals is practically zero. If a series load becomes shorted, the resistance of the other loads keeps the circuit resistance from dropping to zero.

Summary

The important information in this chapter is summarized below.

DC Circuit Faults Summary

- An open series DC circuit will result in no power being consumed by any of the loads.
- The effect of an open in a parallel circuit is dependent upon the location of the open.
- A shorted DC circuit will result in a sudden and very large increase in circuit current.